

TOWARDS INTELLIGENT MATERIALS TESTING WITH REDUCED EXPERIMENTAL EFFORT FOR HOT FORMING

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Abstract

Hot forming processes are typically used to deform metals to the desired shape at lower forming forces and to control the microstructure. During hot deformation, the microstructure evolves by dynamic recrystallization after certain critical conditions are reached. The final recrystallized grain size controls the post-forming mechanical properties of metals and components. To predict the evolution of microstructure and flow stress, various material models were developed and implemented in finite element codes. They require a significant number of material-dependent parameters. Currently, experimental designs with a full-factorial approach for a range of temperature and strain rates are utilized to determine the desired parameters, which involves a huge experimental effort. The aim of this paper is to propose a methodology for parameter identification with reduced experimental effort where progression of testing and data evaluation is parallelized. An iterative, sequential approach is presented which optimizes the new testing conditions based upon preceding experimental conditions. The approach is exemplified for the high-temperature material Alloy-800H, using a material model that allows for accurate predictions of the flow stress. The developed strategy allows to achieve the desired accuracy of the material model by utilizing about a half of test matrix representing a full-factorial design. Hence, an efficient cost- and resource-optimized parametrization of models seems possible.

Key words: hot working, material models, design of experiments, parameter identification

1. INTRODUCTION

Hot working processes are often used to manufacture high-performance metallic components such as turbine blades, gear wheels, etc. In hot working, finite element simulations can contribute to reduce the process development time, the early identification of possible problems or the reduction of material usage (e.g. billet size in closed die forging). Therefore, simulations facilitate a basic understanding of the influence of process parameters on product properties as well as a cost- and resource-efficient production. In this regard, virtual process

design has become a crucial factor for securing competitiveness, especially for mass production and very large forgings where expensive materials such as nickel-base alloys are used.

In order to design, analyze or improve hot working process using finite element simulations, it is vital to use a constitutive model that describes the material response to external loads as accurately as possible. In hot working, the microstructure evolves primarily by recovery, recrystallization and grain growth. These processes change the flow stress which determines important target values such as force or power requirements of the respective hot working process. Hence, the constitutive models

predict both the evolution of microstructure and flow stress of the material at hand.

To model the flow behavior, semi-empirical and physically based models can be distinguished. Semi-empirical flow stress formulations were developed in the past e.g. Sellars (1979), Beynon and Sellars (1992), Senuma et al. (1992), and Laasraoui and Jonas (1991). Simultaneously, physically-based material models were developed by Stüwe and Ortner (1974), Sandström and Lagneborg (1975) as well as Roberts and Ahlblom (1978). In contrast to semi-empirical models, they express the macroscopic flow stress as a function of internal state variables with a physical meaning. The internal state variables (e.g. dislocation density, grain and subgrain size, etc.) represent the microstructural properties of the material. Both semi-empirical and physically based models can be formally written in the following compact notation:

$$\sigma_f = f(\varepsilon, \dot{\varepsilon}, T, \beta_\sigma, \mathcal{S}); \quad \dot{\mathcal{S}} = g(\varepsilon, \dot{\varepsilon}, T, \beta_s, \mathcal{S}) \quad (1)$$

where σ_f is the flow stress, which is a function of strain rate $\dot{\varepsilon}$, the temperature T , material-specific parameters β_σ and the microstructure which is represented by a set of variables \mathcal{S} such as grain size and recrystallized volume fraction or dislocation density. The evolution of the microstructure variables \mathcal{S} depends on the deformation conditions $(\varepsilon, \dot{\varepsilon}, T)$, the current microstructure \mathcal{S} and other additional parameters β_σ . Besides the basic representation mentioned here, the microstructure can be represented in different ways, i.e. a tree or matrix structure can be build up to represent different substructures that are created during recrystallization and may independently evolve successively (Lohmar et al., 2013).

All models draw upon fitting parameters which need to be calibrated from experiments. Although parts of the model, e.g. the recrystallization kinetics, may be formulated based upon physical mechanism, the model parameters are still used as fitting parameters.

To calibrate a material model that couples flow stress and microstructure evolution, accurate experimental data from hot cylindrical compression tests are needed to determine β_σ and β_s . The design of experiments is typically performed a priori. The testing conditions are typically represented by a matrix of testing temperatures and strain rates, defined based upon experience or desired range of the hot forming process. Typically, temperatures between 700-1200°C and strain rates between 0.001-

100/s are tested with step-size of 50°C to 100°C and logarithmic sampling of strain rates. Hence, a full-factorial design with 30-60 individual testing conditions is used. The experimental data form the basis for non-linear regression to identify the model parameters.

This method has various drawbacks. First, it is not known a priori which testing conditions should be used to derive a model of maximum accuracy using a minimum number of tests. Second, some individual tests may produce outliers compared to the remaining experimental data, requiring the test to be repeated. As a consequence, only after a cumbersome testing and evaluation procedure it will be known whether the chosen testing conditions were sufficient and the data are consistent or not.

In a previous work by the first author of this paper, the experimental effort was reduced by leaving out testing conditions from a full-factorial test matrix (Krämer et al., 2015). The results showed that the reduction of experimental effort is feasible, but still this is only known after testing. The aim of the present paper is to explore a more intelligent way of materials testing, where the progression of testing and data evaluation is parallelized. An iterative, sequential approach is presented, which automatically determines new testing conditions based on knowledge from the recorded data (figure 1). As test material, the high-temperature material Alloy 800H has been chosen due to its high material cost.

The aim of devising a more intelligent procedure for parameter identification can only be successfully pursued if an appropriate material model is chosen, i.e., if the model accurately describes the material behavior in the desired range of temperatures and strain rates. This is not a trivial task. On one hand, there is no unique material model for hot working; on the other hand, many existing models show inconsistencies. For dynamic recrystallization, for instance, it is common to use Avrami kinetics (Jonas et al., 2009) for the recrystallized volume fraction. The coefficients of the Avrami equations can be determined from Avrami plots. However, it was shown recently by Bambach (2013a and 2013b) that Avrami kinetics are inconsistent with the second derivative criterion (SDC, see Poliak and Jonas, 1996) if the Avrami exponent does not exceed a value of 3. The SDC is a thermodynamic criterion that marks the onset of DRX. It is important to incorporate these bounds into parameter identification or to find a more consistent model formulation. The latter has been accomplished in Bambach (2013b)



where DRX kinetics and a new strain hardening model was proposed that are consistent with this criterion. This model is used subsequently in this paper.

The paper is organized as follows: section 2 outlines the experimental data and the material model used. The new procedure is presented in section 3. The results and discussion are detailed in section 4 followed by a summary and outlook in section 5.

2. EXPERIMENTAL TESTING AND MATERIAL MODEL

Test data. To analyze the potential for the reduction of the experimental effort the classical approach of a full experimental test matrix is chosen. Flow stress data for Alloy-800H in the temperature range from 900 to 1200°C and strain rates from 0.01 to 10/s, recorded by cylindrical compression testing, is used. The tests were conducted under almost frictionless conditions using Rastegaev samples, compensated for adiabatic heating and smoothed from noise using a moving average filter. This data processing is necessary to obtain suitable data for the fit procedure.

Material model. The experimentally determined flow curves were used to fit the material model proposed in Bambach (2013b). The strain hardening formulation of this model is formulated in Kocks-Mecking space, i.e. in terms of strain hardening rate

Table 1.

Model part	Model equation
Strain hardening	$\theta(\sigma) = \theta_{III}(1 - H_1(\sigma - \sigma_{IV})) + \theta_{IV}H_1(\sigma - \sigma_{IV}) + \theta_V H_2(\sigma - \sigma_V)$ $\theta_{IV} = b_{IV} - m_{IV}\sigma$ $\theta_V = -C(\sigma - \sigma_c)^3$ $H_{1,2}(x) = \frac{1}{2} + \frac{1}{2} \tanh(c_{(1,2)}x) = \frac{1}{1 + e^{-2c_{(1,2)}x}}$
Characteristic stresses	$\sinh(f_3 \cdot \sigma) = f_1 \cdot Z^{f_2}$
DRX kinetics	$\dot{X}(t) = (1 - X(t)) \left[\frac{(1 - H(t_{ss}, t_s))}{3} \frac{4\pi}{I_s} S v^3 t^3 + H(t_{ss}, t_s) A X(t) \right]$
Flow stress	$\hat{\sigma} = \begin{cases} \sigma_0 & \text{if } \varepsilon < \varepsilon_c \\ (1 - X)\sigma_0 + X\sigma_1 & \text{if } \varepsilon \geq \varepsilon_c \end{cases}$

as a function of stress. It is necessary to integrate the model in order to obtain the true stress – true strain curve. The model also draws upon recrystallization kinetics which first models the nucleation of recrystallized grains on prior grain boundaries until site saturation is achieved. In this regime, $\dot{X} \sim 1 - X$ holds. Subsequently, the kinetics switches to a nucleation mode where new recrystallized grains are generated where $\dot{X} \sim X(1 - X)$ is a reasonable approximation for the progression of DRX. A switch function $H(\cdot; \cdot)$ accounts for the transition starting from the point of site saturation t_{ss} . The model equations are given in the table 1.

3. ITERATIVE PARAMETER IDENTIFICATION

The basic idea followed with the proposed procedure is visualized in figure 1. The iterative procedure starts with a number of initial testing conditions and keeps adding tests until reaching a stopping criterion. New test conditions are proposed based on a measure of maximum expected gain.

To determine whether another experiment may improve the model, typically optimal experimental design methods are used for this purpose. These methods draw upon the Fisher information matrix (Stewart, 1987):

$$f_{ij} = \sum_{k=1}^n \frac{1}{\sigma_m^2} \frac{\partial \sigma_f(\varepsilon_k, \dot{\varepsilon}_k, T_k, \beta)}{\partial \beta_i} \frac{\partial \sigma_f(\varepsilon_k, \dot{\varepsilon}_k, T_k, \beta)}{\partial \beta_j} \quad (2)$$

where σ_m is the standard deviation of the measured flow stress.

Here, a different approach is pursued. The main reason for departing from optimal experimental design methods in the application at hand is that typically, no model can be found that predicts the course of flow stress over a broad range of temperatures and strain rates accurately enough, at least the accuracy is not known before experiments are conducted.

The problem is hence not only to find a suitable set of testing conditions to fit a given model but also to find an appropriate model. To tackle both problems at the same time, the following approach is taken.

At first, a small number of experiments are drawn from the full test matrix, using the corner points of the temperature and strain rate matrix and one or more interior points. This small number of flow curves is used to fit the model in two ways using a regularized least squares algorithm.



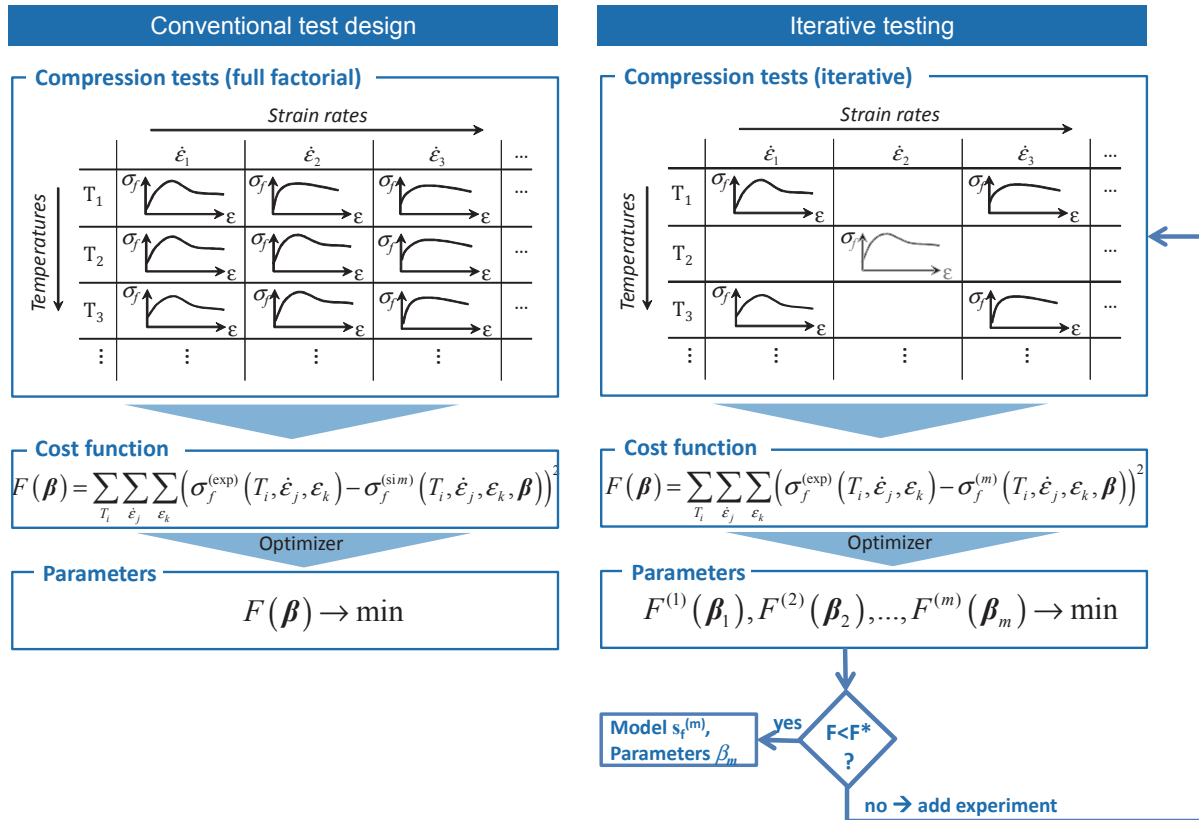


Fig. 1. Conventional and iterative method of materials testing and model calibration.

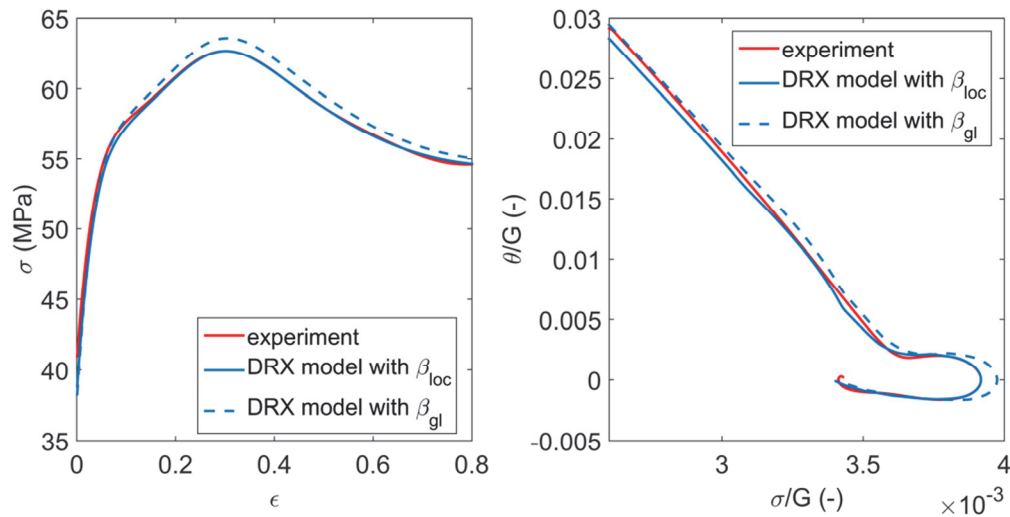


Fig. 2. Curve fitting results from fitting the model to all available flow curves (dotted, global solution) and to a single curve (solid, local solution). The comparison is shown for $T=1200^\circ\text{C}$ and a strain rate of $0.1/\text{s}$. The global parameter set produces larger deviations on the flow curve than the one fitted to this flow curve only.

First, the model is fitted to all currently available flow curves. If this is not possible with a desired accuracy with a deviation of less than 10%, the model is fitted to all flow curves individually using the global solution as a starting guess. Due to the fact that the model is designed to reproduce the course of the strain hardening rate as a function of stress, it will be able to approximate each individual

flow curve very accurately even though it may not be possible to find a single parameter vector to describe all flow curves together with the desired accuracy. From these fitting procedures, a global parameter vector $\boldsymbol{\beta}_{gl}$ and local parameter vectors $\Delta\boldsymbol{\beta}$ are obtained. The parameter vector hence can be expressed as:



$$\boldsymbol{\beta}(r, T) = \boldsymbol{\beta}_{gl} + \Delta\boldsymbol{\beta}(r, T) \quad (3)$$

where the strain rate $\dot{\epsilon}$ has been denoted r , which will enable an unambiguous notation when the strain rate is used as variable in the following.

The fluctuations of the parameter vector are approximated by an interpolation model using radial basis functions (RBF), for which a fast solution algorithm was presented in earlier work (Grzhibovskis et al., 2008). Thin plate splines have been chosen among the different RBF kernels in the present paper, as they minimize the bending energy of the interpolating function, i.e. like cubic splines, they try to fit the data in such a way that the curvature of the resulting surface is minimized. To obtain an explicit representation $\Delta\boldsymbol{\beta}(r, T)$ of the fluctuation of the parameter vector as a function of strain rate and temperature, the following minimization problem has to be solved for each component $\Delta\beta_j$ of the parameter vector:

$$\frac{1}{n} \sum_{i=1}^n (\Delta\beta_j(r_i, T_i) - \Delta\beta_j)^2 + \lambda \sum_{j=1}^m \binom{m}{j} \iint_{\mathbb{R}^2} \left(\frac{\partial^m \Delta\beta_j(r_i, T_i)}{\partial^j \partial^{m-j}} \right)^2 drdT \rightarrow \min \quad (4)$$

where λ is a regularization parameter that describes the trade-off between interpolation and approximation. For a C^2 -continuous solution ($m = 2$), $\Delta\boldsymbol{\beta}(r, T)$ is of the form:

$$\Delta\beta_j(r, T) = \sum_{i=1}^N a_i \phi_i(r, T) + a_{N+1}r + a_{N+2}T + a_{N+3} \quad (5)$$

where the TPS kernel

$$\phi_i(r, T) = \left(\|(r, T) - (r_i, T_i)\|_2 \right)^2 \ln \|(r, T) - (r_i, T_i)\|_2 \quad (6)$$

corresponds to the fundamental solution of the bi-harmonic (plate) equation up to a constant factor. The unknown coefficients, a_i , solve the following system of $n + 3$ linear equations:

$$\begin{pmatrix} \mathbf{A} + n\lambda\mathbf{I} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{pmatrix} \underline{a} = \underline{z} \quad \text{with} \quad \mathbf{A} = (\phi_i(r_j, T_j))_{i,j=1}^n$$

$$\text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & r_1 & T_1 \\ \vdots & \vdots & \vdots \\ 1 & r_n & T_n \end{pmatrix} \quad (7)$$

where $\underline{a} = (a_1, \dots, a_{n+3})^T$ is the solution vector and

$\underline{z} = (\Delta\beta_1, \dots, \Delta\beta_n, 0, 0, 0)^T$ is the right hand side.

Using this local correction of the model, each flow curve can be approximated with suitable accuracy. If the fluctuations turn out to be small, the part $\Delta\boldsymbol{\beta}$ may be dropped eventually.

With a model of suitable accuracy, it is possible to proceed developing a greedy algorithm that iteratively increases the number of testing conditions until the model reaches suitable accuracy on the entire range of testing conditions. To decide whether another experiment is needed and if so, at which temperature and strain rate, an approximant of the available set of flow curves is created and compared to the model. Let n flow curves $\sigma_{f,i}(\epsilon, \dot{\epsilon}_i, T_i)$, where $i=1..n$, be given. If the information contained in the available flow curves is used to build an approximation of the flow stress on the entire range of temperature and strain rate, the distance between the approximant and the model built upon the available test data can be determined for all temperatures and strain rates in the desired range.

This approximant is created again using the thin plate spline approximation. From the comparison of the TPS approximant $\tilde{\sigma}_n(\epsilon, \dot{\epsilon}, T)$ and the model, $\hat{\sigma}_n(\epsilon, \dot{\epsilon}, T)$, the point of maximum deviation is obtained:

$$(\dot{\epsilon}_{i^*}, T_{j^*}) = \arg \max \left\| \tilde{\sigma}_n(\epsilon, \dot{\epsilon}_i, T_j) - \hat{\sigma}_n(\epsilon, \dot{\epsilon}_i, T_j) \right\| \quad (8)$$

If the deviation at this point is larger than the desired threshold, the point of maximum deviation is taken as new test condition; another test is performed and added to the data basis. Then, the approximant and model are updated and again compared. This procedure is repeated until the threshold is no longer exceeded on the entire range of testing conditions, i.e. until the sequence $F^{(m)}(\boldsymbol{\beta})$ of cost functions converges to a desired value.

4. RESULTS AND DISCUSSION

The iterative procedure is visualized in figure 2. It converges to a deviation of less than 10% using only 20 of the 35 flow curves given by all grid points. Compared to the full-factorial test design, the experimental effort is reduced considerably.

It is shown that a little more than half of the test matrix representing a full-factorial design is sufficient to reach the desired accuracy comparable to using the full matrix.



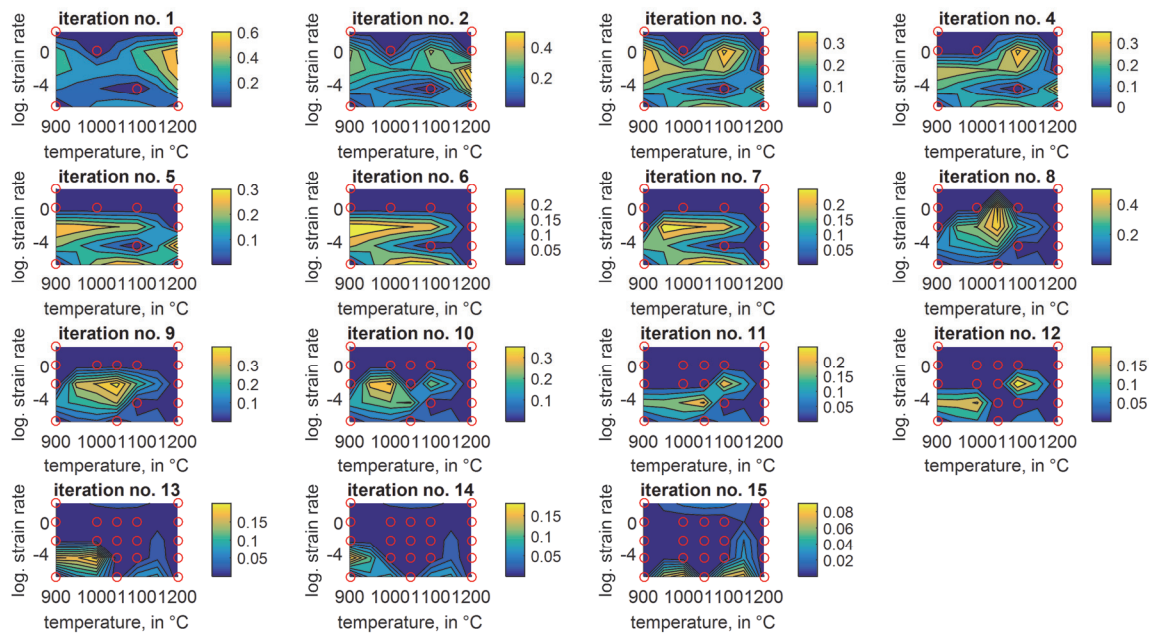


Fig. 3. Convergence of the iterative method to a model error of less than 10% in 15 iterations.

The method has been applied off-line to test whether it is feasible to turn the conventional approach of parameter identification into an iterative scheme. The most severe problem at the moment seems to be the identification of a suitable model, since optimal testing conditions and model are strongly coupled. Since quite a large number of models were developed in the past, the implementation on a testing machine for automated parameter evaluation could be performed by evaluating several models in competition, eliminating step by step those models which fall behind the more suitable ones. However, testing conditions are unlikely to be optimal for all considered models. Hence, a large effort for fitting all models in parallel would be required, and some models may be eliminated as a consequence of wrong testing conditions used for that particular model.

The presented approach does not suffer from such drawbacks. Provided that the global and local solutions are sufficiently close, the global solution may be applied for most testing conditions, and the local correction may be used only in certain ranges of temperature and strain rate.

The model in its present state excludes the evolution of grain size which is not measured directly during the compression tests but only in cumbersome subsequent metallography. To extend the method for microstructural features, on-line measurement of grain size, e.g. using laser ultrasonic determination, would be helpful.

Implementation on a testing machine also requires that the inaccuracies of the compression test

due to inhomogeneity of temperature and frictional effects are taken into account. While compensation methods for adiabatic heating and friction are available, their accuracy and robustness needs to be analysed in order to automate materials testing with the proposed method. Alternatively, inverse methods could help to deal with experimental inaccuracies.

Also, the force-displacement data resulting from the test should be processed into flow stress data without artifacts. Due to the fact that various filtering and smoothing steps are involved, this task may be a source for erroneous results when the presented method is automated.

5. CONCLUSIONS AND OUTLOOK

A method for iterative, more intelligent parameter identification of flow stress models for hot working was presented. The following conclusions can be drawn:

- Automatic selection of testing conditions for parameter determination of flow stress models seems viable. However, the prerequisite is that the model represents the experimental data well enough over a large range of testing conditions and that experimental sources of uncertainty and inaccuracy are handled.
- Due to this requirement, a procedure has been presented that updates both the testing conditions and the model, by splitting the parameter vector into a global average and local fluctuations.



- With a model of suitable accuracy, a greedy iterative procedure was implemented that keeps adding additional tests until the model reaches the desired threshold accuracy on the entire matrix of testing conditions.
- The accuracy of the model under unknown testing conditions is estimated by comparing the model with a thin plate spline interpolant of the test data. This comparison yields the estimated deviation on the entire test grid and allows for identifying the next testing condition if another test is necessary.

Future work will focus on implementing the method on a deformation dilatometer. Due to its possibility to heat specimens by induction under vacuum or protective atmosphere, each test can be performed quickly and evaluated directly after testing, so that new testing conditions can be determined while the next test is set up.

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OBNIŻENIE KOSZTÓW DOŚWIADCZEŃ DLA PLASTYCZNEJ PRZERÓBKII NA GORĄCO POPRZEZ ZASTOSOWANIE INTELIGENTNYCH BADAŃ MATERIAŁÓW

Streszczenie

Procesy plastycznej przeróbki na gorąco są zazwyczaj wykorzystywane zarówno do nadawania wymaganego kształtu wyrobom przy zastosowaniu mniejszych sił jak i do kontrolowania ich mikrostruktury. Mikrostruktura w czasie odkształcania na gorąco zmienia się pod wpływem rekrytalizacji dynamicznej, która występuje po osiągnięciu pewnych krytycznych warunków. Wielkość ziarna po rekrytalizacji wyznacza własności mechaniczne wyrobu po kształtowaniu. Aby przewidywać rozwój mikrostruktury i wielkość naprężenia uplastyczniającego opracowane zostały różne modele, które zaimplementowano w programach z metody elementów skończonych. Te modele wymagają wyznaczenia szeregu parametrów charakterystycznych dla danego materiału. Klasycznym podejściem do wyznaczenia tych parametrów jest przeprowadzenie pełnego cyklu badań w szerokim zakresie temperatur i prędkości odkształcania, co jest to podejściem bardzo kosztownym. Dlatego za cel pracy postawiono sobie opracowanie metodologii identyfikacji parametrów modelu materiału na podstawie zredukowanej liczby doświadczeń, wykorzystując zrównoleżenie oceny danych. W artykule opisano iteracyjną, sekwencyjną procedurę optymalizującą warunki kolejnych prób doświadczalnych na podstawie wyników prób wcześniejszych. Jako przykład zastosowania tej procedury wybrano odkształcanie stopu 800H w wysokich temperaturach. Identyfikowano model prawidłowo opisujący naprężenie uplastyczniające tego stopu. Zaproponowana strategia pozwoliła uzyskać wymaganą dokładność modelu wykorzystując połowę prób wynikających z pełnego planu eksperymentów. Przeprowadzona analiza potwierdziła możliwość wydajnej kosztowo identyfikacji modeli.

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