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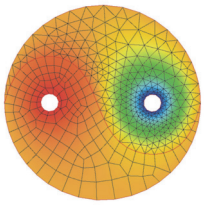

OPTIMIZATION OF GEOMETRICAL STRUCTURES OF MODERN MATERIALS BY USING A HYBRID EVOLUTION STRATEGY

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Abstract

In this paper, a new hybrid evolutionary-gradient algorithm is proposed to solve global numerical problems of topology optimization. The proposed algorithm is developed as a compromise between the direct and gradient-based optimization approaches. A short comparative study of computational results demonstrates the validity of the proposed method, as well as the necessity of its use.

Key words: topology optimization, gradient optimization algorithms, hybrid evolution strategy

1. INTRODUCTION

One of the most fundamental areas of materials science is development of mechanical material properties. In recent years, artificial materials engineered to have properties, which may not be found in nature. Generally, they have extraordinary effective properties, such as negative dynamic modulus and/or density, superior thermoelectric properties, phononic bandgaps and high specific energy absorption. These metamaterials are usually arranged in repeating patterns and are assemblies of multiple individual elements fashioned from conventional microscopic materials such as metals or plastics. Their extraordinary properties are created not by the choice of their chemical composition, but by their exactingly-designed geometric structures constituted by precise shape, size, orientation and arrangement.

Geometrical structures of modern materials can now be much more controlled in order to design the structural anisotropy, which determines the deformation pattern and directions of lines of action of the internal forces. Thus by fixing the structural anisotropy, geometrical structures of modern materials could be designed to create complex stress-strain paths to protect a certain internal volume. Natural and produced artificially co-continuous structures, which include hard and soft materials, can provide superior combinations of properties such as stiffness, strength, impact resistance, toughness, and energy dissipation. Optimizing topology and geometric arrangement of components of such a structure provides the possibility of obtaining the combination of desired properties in the macroscale. The topology optimization techniques can be applied, among others, to designing materials at micro and nano-levels. Numerical methods needed to carry out an effective topology optimization of continuum structures have been investigated extensively. Most of these methods are established on the basis of finite and/or boundary element analysis, where the design domain is discretized into a fine mesh of elements.

For a given set of loads and boundary conditions, the primary goal of the topology optimization

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procedure is to find the topology of a structure by determining for every point in the design domain whether the specific point of space become a part of the material matrix or the void space. Using the optimization of topology, namely, the optimization of such properties of geometrical shapes that remain the same even when the shapes change, engineers can find the best concept design, which meets the design requirements for performance and manufacturability. Rather than limiting the changes to the sizes of structural components, topology optimization provides much more freedom and allows the designer to create totally novel and highly efficient conceptual designs for continuum structures. Since topology optimization of continuum structures is the most difficult technically as well as the most advantageous economically in comparison with other types of structural optimization, further publication sections will be devoted to this issue.

2. FORMULATION OF A STRUCTURAL OPTIMIZATION PROBLEM (SOP)

In stating any structural constrained optimization problem the objective function (f), which value indicates the goodness of the design is always introduced together with the following variables (Christensen & Klarbring, 2009):

- Design variables (collected in a vector **x**) that describe the design (i.e. geometrical shape or choice of material), and which can be changed during optimization.
- State variables (collected in a vector **y**) that represent the responses of the structure for a given design x (note that displacements, stresses, strains, forces or vibrations can be assigned to "structural responses").

Hence, a general structural optimization problem (**SOP**) can be stated as follows:

$$\textbf{SOP}\begin{cases}\text{minimize or maximize } f(\mathbf{x},\mathbf{y}) \text{ with respect to } \mathbf{x} \text{ and } \mathbf{y}\\ \text{subject to}\begin{cases}\text{behavioral constrains on } \mathbf{y},\\ \text{design constrains on } \mathbf{x},\\ \text{equilibrium constraints.}\end{cases}\end{cases} \quad (1)$$

In structural optimization all kinds of requirements due to the function $f(\mathbf{x},\mathbf{y})$ that act as constraints can be expressed by

- Inequalities:
$$\forall j = 1,\dots,\mathrm{r}\ \text{(for r pasive constrains)} \quad g_j(\mathbf{x},\mathbf{y}) \le \bar{g}_j\ ,$$
- Equalities:
$$\forall j = \mathrm{r}+1,\dots,\mathrm{s}\ \text{(for s-r active constrains)} \quad g_j(\mathbf{x},\mathbf{y}) = \bar{g}_j\ .$$

In addition there can be constraints on the variables x and y themselves, and they are defined as

$$\check{\mathbf{x}} \le \mathbf{x} \le \hat{\mathbf{x}} \qquad \text{and} \qquad \check{\mathbf{y}} \le \mathbf{y} \le \hat{\mathbf{y}} \quad (2)$$

or defined by means of subsets of **x** and **y** with assigned (fixed) values. The process of solving a system with continuously distributed parameters requires a discretization, which produces a discrete parameter system modeled with a finite element method (**FEM**) or a boundary element method (**BEM**). Note that a **FEM** mesh (or a **BEM** mesh) is only fixed during a given iteration step of the sequential approximate optimization. For static optimization problems discretized by means of the **FEM**, equilibrium constraints take the form:

$$\mathbf{K}(\mathbf{x}) \cdot \mathbf{U} = \mathbf{F}(\mathbf{x}) \quad (3)$$

where **K**(**x**) is the stiffness matrix of the structure, which is a function of the design variables, **U** is the displacement vector and **F**(**x**) is the static force vector, which is also depend on the design variables. Note that displacements (collected in a vector **U**) take the role of general state variables.

A multi-objective optimization problem (**MOOP**) deals with several objective functions (collected in an optimization vector **f**), such that

$$\text{minimize}_{\mathbf{x},\mathbf{y}} \left\{ f_1(\mathbf{x},\mathbf{y}), f_2(\mathbf{x},\mathbf{y}), \dots, f_l(\mathbf{x},\mathbf{y}) \right\} \quad (4)$$

where l (>1) is the number of objective functions. It is worth to stress that the requirements due to the objective functions (implemented in the forms of constraints) are the same as for the **SOP**. Note that tasks of minimization and maximization of objective functions are trivially related to each other, since one objective function $f_k(\mathbf{x},\mathbf{y})$ ($\forall k = 1,2,\dots,l$) could just as well be converted into another function with minus sign: $-f_k(\mathbf{x},\mathbf{y})$.

To handle with problems of multi-objective optimization, the Pareto optimal solutions are sought. A structural design is Pareto optimal, if there is no any other design that satisfies all of the objectives better. Thus, the meaning of Pareto optimal solution is as follows: $(\mathbf{x}^*,\mathbf{y}^*)$ satisfying taken constraints is Pareto optimal, if there is no other feasible solution $(\mathbf{x},\mathbf{y})$ satisfying the constraints, which may combine an value

decrease of at least one objective function without value increases of the remaining objective functions in comparison to the solution $(\mathbf{x}^*, \mathbf{y}^*)$, and what can be mathematically written as

$$\exists k \in \{1,2,\ldots,\mathrm{l}\}: f_k(\mathbf{x}^*,\mathbf{y}^*) < f_k(\mathbf{x},\mathbf{y}) \quad \wedge \quad \forall k = 1,2,\ldots,\mathrm{l}: f_k(\mathbf{x}^*,\mathbf{y}^*) \leq f_k(\mathbf{x},\mathbf{y}). \tag{5}$$

In multi-objective optimization problems, it is characteristic that no unique solution exists but a set of mathematically equivalently good solutions, which can be described as nondominated, efficient and noninferior.

The method of Lagrange multipliers can be applied to identify candidate optimal points of equality-constrained optimization problems. If functions: f (objective or performance) and $\mathbf{g}$ (constraints) are entirely functions of m design variables $(x_1, x_2, \ldots, x_\mathrm{m})$, then the constrained optimization problem corresponding to only active constraints can be reformulated with a Lagrangian function $\mathcal{L}$ as

$$\mathcal{L}(\mathbf{x},\boldsymbol{\lambda}) = f(\mathbf{x}) + \sum_{j=1}^{\mathrm{p}} \lambda_j \left[g_j(\mathbf{x}) - \bar{g}_j \right], \tag{6}$$

where the λ_j are the Lagrange multipliers corresponding to the active constraints. Let us assume that the stationary point of Lagrangian function $\mathcal{L}$ corresponds to the local minimum. Partial differentiation of $\mathcal{L}$ with respect to the design variables gives

$$\frac{\partial \mathcal{L}(\mathbf{x},\boldsymbol{\lambda})}{\partial x_i} = \frac{\partial f(\mathbf{x})}{\partial x_i} + \sum_{j=1}^{\mathrm{p}} \lambda_j \frac{\partial g_j(\mathbf{x})}{\partial x_i}. \tag{7}$$

Necessary conditions for $\mathbf{x}^*$ to be a local minimum of $\mathcal{L}$ on the intervals $x_i^{\min} \leq x_i \leq x_i^{\max}$ $(\forall i = 1,2,\ldots,\mathrm{m})$ are that

$$\left. \frac{\partial \mathcal{L}(\mathbf{x},\boldsymbol{\lambda})}{\partial x_i} \right|_{x_i = x_i^*} = 0. \tag{8}$$

Conditions (8) are necessary, what means that if they are not satisfied, then $\mathbf{x}^*$ is not a local minimum. Assuming that the Lagrangian function $\mathcal{L}$ is strictly convex, a unique solution to the minimization problem may be derived on the basis of the Karush–Kuhn–Tucker stationary conditions (Christensen & Klarbring, 2009).

3. NUMERICAL METHODS FOR TOPOLOGY OPTIMIZATION

It seems, that there are two main directions of development of topology optimization, which are referred to the following methods: the evolutionary structural optimization (**ESO**) method and the solid isotropic microstructure with penalization (**SIMP**) method (Rozvany, 2009). The **ESO** method is a design method based on the simple concept of gradually removing the inefficient, low-stressed material from a structure by deleting elements with the lowest strain energy density from the finite element model. The idea of the **ESO** method has developed from the simple one-directional optimization schemes to the bi-directional evolutionary structural optimization (**BESO**) method, which allows to remove solid elements with the lowest strain energy density from the structure and to change the void elements with the highest strain energy density into solid elements, simultaneously. The initial research on **BESO** was conducted by Young et al. (1999) for stiffness optimization. According to the **SIMP** approach the variable density is used to do the topology optimization. In the optimization process each finite element e is assigned to the density ρ_e, which determines its Young's modulus E_e as stated in the following formula:

$$E_e(\rho_e) = \mathrm{E}_{\min} + \rho_e^{\mathrm{p}}(\mathrm{E}_0 - \mathrm{E}_{\min}) = \left[\gamma + (1-\gamma)\rho_e^{\mathrm{p}}\right] \mathrm{E}_0, \tag{9}$$

where E_0 is the real Young's modulus of the material, $\mathrm{E}_{\min}$ is a very small value of the Young's modulus, which is assigned to void regions in order to prevent the stiffness matrix from becoming singular, p is a penalization factor (a given value usually between 2 and 4 to penalize the small values of ρ_e, typically $\mathrm{p} = 3$ acc. Andreassen et al. (2011)) and $\gamma \stackrel{\text{def}}{=} \mathrm{E}_{\min}/\mathrm{E}_0$. To obtain a solid-void design with maximum stiffness, the mean compliance is minimized for a given volume of material. The topology optimization problem can be stated as

$$\min_{\boldsymbol{\rho}} f(\boldsymbol{\rho}) = \mathbf{U} \cdot \mathbf{K} \cdot \mathbf{U} = \sum_{e=1}^{\mathrm{n}} \left[\gamma + (1-\gamma)\rho_e^{\mathrm{p}}\right] \mathbf{u}_e \cdot \mathbf{k}_e \cdot \mathbf{u}_e, \tag{10a}$$

$$\text{subject to} \quad \begin{cases} g_0(\boldsymbol{\rho}) - \bar{g}_0 = \frac{V(\boldsymbol{\rho})}{\mathrm{V}_0} - \mathrm{f_V} = \\ \frac{1}{\mathrm{V}_0} \sum_{e=1}^{\mathrm{n}} v_e \, \rho_e - \mathrm{f_V} = 0 \\ \mathbf{K} \cdot \mathbf{U} = \mathbf{F}, \\ \mathbf{0} < \boldsymbol{\rho}_{\min} \leq \boldsymbol{\rho} \leq \mathbf{1}, \end{cases} \tag{10b}$$

where $\mathbf{u}_e$ is the element displacement vector, $\mathbf{U}$ and $\mathbf{F}$ are the global displacement and force vectors, respectively, $\mathbf{K}$ represents the global stiffness matrix, while $\mathbf{k}_e$ is the element stiffness matrix for the e-th element with material, which has the Young's modulus E_0. There are n finite elements used to dis-

cretize the design domain, with the corresponding vector of elemental densities $\boldsymbol{\rho}=\{\rho_1,\rho_2,\ldots,\rho_n\}$ (i.e., the vector of the design variables). Furthermore, $V(\boldsymbol{\rho})$ represents the material volume, v_e the volume of the e-th element, V_0 the total volume of the design domain, f_V a prescribed volume fraction and the mark "$\cdot$" means the inner product.

In order to solve the topology optimization problem (10) Bendsøe (1995) has proposed the following heuristic updating scheme for the design variables:

$$\rho_e^{\text{new}} = \begin{cases} \rho_e B_e^{1/\alpha} & \text{if} \quad \check{\rho}_e < \rho_e B_e^{1/\alpha} < \hat{\rho}_e\,, \\ \check{\rho}_e & \text{if} \quad \rho_e B_e^{1/\alpha} \le \check{\rho}_e\,, \\ \hat{\rho}_e & \text{if} \quad \rho_e B_e^{1/\alpha} \ge \hat{\rho}_e\,, \end{cases} \qquad \forall\, e = 1,2,\ldots,n, \text{with} \tag{11a}$$

$$\check{\rho}_e \leftarrow \max\left(\rho_{\min}, \rho_e - \delta\right), \quad \hat{\rho}_e \leftarrow \min\left(\rho_e + \delta, 1\right), \tag{11b}$$

where $\delta>0$ is a prescribed move limit. Then, the algorithm (11) is iteratively solved until the following stopping condition is satisfied:

$$\| \boldsymbol{\rho}^{\text{new}} - \boldsymbol{\rho} \| < \epsilon_b \tag{11}$$

with $\epsilon_b>0$ and prescribed. In the updating scheme (11), the B_e are found from the stationary conditions (8) as:

$$B_e = -\left(\lambda \frac{\partial g_0(\boldsymbol{\rho})}{\partial \rho_e}\right)^{-1} \frac{\partial f(\boldsymbol{\rho})}{\partial \rho_e} \tag{13}$$

During applying the algorithm (11), the Lagrange multiplier may be found using a simple bisectioning strategy (although more advanced strategies are also possible). The first derivatives of the compliance defined as $C(\boldsymbol{\rho})=\mathbf{F}\,\mathbf{U}$ ($\stackrel{\text{def}}{\equiv} f(\boldsymbol{\rho})$) with respect to the design variables ρ_e ($\forall e=1,2,\ldots,n$) can be obtained by the following derivations:

$$\frac{\partial C(\boldsymbol{\rho})}{\partial \rho_e} = \mathbf{F}\cdot\frac{\partial \mathbf{U}}{\partial \rho_e} \;\because \text{(a) } \mathbf{F} \text{ is the static force vector, and (b) } \mathbf{F} \text{ is independent of } \rho_e, \tag{14}$$

and moreover, $\partial\mathbf{U}/\partial\rho_e$ is to be

$$\frac{\partial \mathbf{U}}{\partial \rho_e} = \frac{\partial(\mathbf{K}^{-1}\mathbf{F})}{\partial \rho_e} = \frac{\partial \mathbf{K}^{-1}}{\partial \rho_e}\cdot\mathbf{K}\cdot\mathbf{U} = -\mathbf{K}^{-1}\cdot\frac{\partial \mathbf{K}}{\partial \rho_e}\cdot\mathbf{U}\,. \tag{15}$$

Substituting (15) into (14) yields

$$\frac{\partial C(\boldsymbol{\rho})}{\partial \rho_e} = -\mathbf{F}\cdot\mathbf{K}^{-1}\cdot\frac{\partial \mathbf{K}}{\partial \rho_e}\cdot\mathbf{U} = -\mathbf{U}\cdot\frac{\partial \mathbf{K}}{\partial \rho_e}\cdot\mathbf{U} = -\mathbf{u}_e\cdot\frac{\partial \mathbf{k}_e}{\partial \rho_e}\cdot\mathbf{u}_e \;\because\; \mathbf{K}^{-1} \text{ is symmetric}\,. \tag{16}$$

To reduce the overall number of iterations needed to attain the optimum the objective and constraint functions from the formulation (10) are replaced with their approximations at the current design point (Fadel et al., 1990). In topology optimization the linear, reciprocal, and exponential approximations of the objective and constraint functions are all based on a truncated first-order Taylor series expansion (Groenwold & Etman, 2008).

Substituting the following exponential intermediate variables:

$$y_e = \left(\frac{z_e - \check{\rho}_e}{\hat{\rho}_e - \check{\rho}_e}\right)^a \quad \left(\forall z_e \in [\check{\rho}_e, \hat{\rho}_e], \text{ where } e = 1,2,\ldots,n\right) \tag{17}$$

into the exponential approximation of the objective function $f(\boldsymbol{\rho})$ described in the formulation (10) about the current design point $\mathbf{z}^0$ gives

$$f_{\text{app}}(\mathbf{z}) = f(\mathbf{z}^0) + \sum_{e=1}^{n} \frac{\partial f}{\partial z_e}\Big|_{\mathbf{z}=\mathbf{z}^0} \frac{1}{a}(z_e^0 - \check{\rho}_e)\left[\left(\frac{z_e-\check{\rho}_e}{z_e^0-\check{\rho}_e}\right)^a - 1\right]. \tag{18}$$

The constraint function $g_0(\boldsymbol{\rho})$ introduced in the formulation (10) about the current design point $\mathbf{z}^0$ is approximated linearly

$$g_{\text{app}}(\mathbf{z}) = g(\mathbf{z}^0) + \sum_{e=1}^{n} \frac{\partial g}{\partial z_e}\Big|_{\mathbf{z}=\mathbf{z}^0} (z_e - \check{\rho}_e)\,. \tag{19}$$

Necessary conditions (8) for $\mathbf{z} = \mathbf{z}^*$ to be optimal provide the relationships between the Lagrange multiplier λ and design variables:

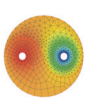
$$\forall e = 1,2,\ldots,n: \quad \frac{\partial f_{\text{app}}}{\partial z_e} + \lambda\frac{\partial g_{\text{app}}}{\partial z_e} = 0\,. \tag{20}$$

By substitution equations (18) and (19) into equation (20) yields:

$$\left(\frac{z_e-\check{\rho}_e}{z_e^0-\check{\rho}_e}\right)^{1-a} = B_e\left(\equiv \frac{-\frac{\partial f}{\partial z_e}\Big|_{\mathbf{z}=\mathbf{z}^0}}{\lambda\frac{\partial g}{\partial z_e}\Big|_{\mathbf{z}=\mathbf{z}^0}}\right). \tag{21}$$

Thus, the explicit expression, which assesses the optimal design variables z_e^* for the next iteration takes the form:

$$z_e^* = \check{\rho}_e + B_e^{\frac{1}{1-a}}(z_e^0 - \check{\rho}_e)\,. \tag{22}$$



It is noteworthy that the **SIMP** and **BESO** methods are general and can handle any type of structural models and objective functions. However, a straightforward implementation of these methods to perform topology optimization in the finite element framework is, in some cases, hampered owing to drawbacks such as: mesh dependency and numerical instabilities. An adequate remedy to these drawbacks seems to be the application of homogenization methods (Bendsøe & Kikuchi, 1988; Allaire, 2002) but they are mainly restricted to linear elasticity and particular objective functions. Therefore, another type of method known as the level-set method (**LSM**), which was first proposed by Sethian and Wiegmann (1999) in structure optimization, is becoming more popular because of its computational efficiency. In structural optimization the **LSM** provides a general framework for the propagation of a material interface between material and void domains by means of a level-set function ϕ, which is a continuous Lipschitz function. For the given function ϕ, the material domain $\Omega^+\backslash\Gamma$, the void domain Ω^- and the material interface Γ inside the design domain Ω are defined as,

$$\begin{cases} \forall\, \mathbf{x} \in \Omega^+\backslash\Gamma \text{ (material)}: & \phi(t,\mathbf{x}) > 0 \\ \forall\, \mathbf{x} \in \Gamma\,(\stackrel{\text{def}}{=} \partial\Omega^+) \text{ (interface)}: & \phi(t,\mathbf{x}) = 0 \\ \forall\, \mathbf{x} \in \Omega^-\,(\stackrel{\text{def}}{=} \Omega\backslash\Omega^+) \text{ (void)}: & \phi(t,\mathbf{x}) < 0. \end{cases} \tag{23}$$

So, the level-set function ϕ is used both to represent and evolve the interface Γ. In the **LSM**, apart from the implicit function ϕ, the most commonly used functions are the Heaviside function $H(\phi)$:

$$H(\phi) = \begin{cases} 0 & \text{if} \quad \phi < 0, \\ 1 & \text{if} \quad \phi \geq 0, \end{cases} \tag{24}$$

and the one-dimensional Dirac delta function $\delta(\phi)$:

$$\delta(\phi) = \frac{dH(\phi)}{d\phi}. \tag{25}$$

A small but arbitrary non-zero change in the ordinate of $H(\phi)$ for a particular value of ϕ within the design domain Ω can be determined as follows:

$$\delta H(\phi) = \frac{dH(\phi)}{d\phi}\,\delta\phi = \delta(\phi)\,\delta\phi|_{\phi=0} \because$$

$$\delta(\phi) = 0 \quad \text{for} \quad \phi \neq 0. \tag{26}$$

Following the optimization process, the shape $\Omega^+(t)$ is going to evolve according to a fictitious time t. To be precise, assume that the shape $\Omega^+(t)$ evolves in time $t \in \mathbb{R}^+$ with a velocity $\mathbf{V}(t)$ ($\stackrel{\text{def}}{=} \dot{\mathbf{x}}(t)$) of each point $\mathbf{x}(t)$ on the boundary $\Gamma(t)$ ($\stackrel{\text{def}}{=} \partial\Omega^+(t)$). Then

$$\forall \mathbf{x}(t) \in \Gamma(t): \quad \phi\big(t, \mathbf{x}(t)\big) = 0\,. \tag{27}$$

Differentiating the above equation in t yields the Hamilton–Jacobi equation:

$$\frac{\partial\phi}{\partial t} + \nabla\phi \cdot \dot{\mathbf{x}} = \frac{\partial\phi(\mathbf{x},t)}{\partial t} + V_{\mathrm{n}}(\mathbf{x},t)\,\|\nabla\phi(\mathbf{x},t)\| = 0 \tag{28}$$

$$\left(\begin{array}{c} \because \text{ tangential components of } \mathbf{V}\ (\stackrel{\text{def}}{=} \dot{\mathbf{x}}) \\ \text{vanish: } \nabla\phi \cdot \dot{\mathbf{x}} = \nabla\phi \cdot (V_{\mathrm{n}}\mathbf{n}) \equiv V_{\mathrm{n}}\,\nabla\phi \cdot \dfrac{\nabla\phi}{\|\nabla\phi\|} \end{array} \right),$$

which governs the evolution of the level-set function ϕ in time. In equation (28) $V_{\mathrm{n}}(\mathbf{x}, t) \stackrel{\text{def}}{=} \mathbf{V}\cdot\mathbf{n}$ ($\stackrel{\text{def}}{=} \dot{\mathbf{x}}\cdot\mathbf{n}$) is the normal velocity function.

By assuming that points of the material interface Γ, which is represented by the zero level set: $\phi(t, \mathbf{x}) = 0$, move only in the normal direction $\mathbf{n}$ ($=\nabla\phi/\|\nabla\phi\|$) to Γ, a small change in a variable quantity $\delta\phi|_{\phi=0}$ due to an arbitrarily small increment (i.e., a virtual change) δl along the normal direction n can be expressed as

$$\delta\phi|_{\phi=0} = \phi(\mathbf{x} + \delta l\,\mathbf{n}) - \phi(\mathbf{x}) \cong$$

$$\nabla\phi \cdot (\delta l\,\mathbf{n}) = \|\nabla\phi\|\,\delta l\,. \tag{29}$$

The Lagrangian method can be utilized to convert the original constrained optimization problem (10) into an unconstrained problem as

$$\mathcal{L}_{\mathrm{LS}}(t,\Omega,\phi,\mathbf{u},\mathbf{v},\lambda) = \int_\Omega [E(\phi)\mathbf{D} : \boldsymbol{\varepsilon}(\mathbf{u})] : \boldsymbol{\varepsilon}(\mathbf{v})\,d\Omega \quad + \quad \lambda\left[\int_\Omega H(\phi)\,d\Omega - \mathrm{f_V V_0}\right], \tag{30}$$

where $\boldsymbol{\varepsilon}$ is the strain field, $\mathbf{u}$ is the actual displacement field, $\mathbf{v}$ denotes the arbitrary virtual displacement field, which displacements are kinematically admissible, λ means the Lagrange multiplier of the volume constraint, $\mathbf{D}$ is the elasticity matrix with unit Young's modulus and $E(\phi)$ indicates the design variable, which is defined by means of the following equation:

$$E(\phi) = \mathrm{E_0}\,H(\phi) + [1 - H(\phi)]\,\mathrm{E_{min}}. \tag{31}$$

In equation (31) $\mathrm{E_0}$ and $\mathrm{E_{min}}$ represent the material elasticity module and the minimum relative elasticity module, respectively. According to the principle of minimum potential energy, the displacement field $\mathbf{u}$ corresponds to the absolute minimum of the potential energy functional.

It is known from the paper by Allaire et al. (2004) that the standard version of the level-set method is in general not capable of creating new holes in the structure during the optimization process. However, the level-set method can change the

topology by closing holes or merging several holes together. To further improve computational efficiency and overcome the difficulty that the standard version of the level-set method cannot generate new holes during the optimization process, it is assumed that the level-set function $\phi(\mathbf{x})$, considering also as phase-field function (Gain & Paulino, 2012), evolves in the direction, which minimizes the objective functional.

It is widely believed (Sigmund, 2011) that for the majority of topology optimization problems the use of non-gradient, nature inspired methods like genetic algorithms is not reasonably practicable. However, the most successful methods nowadays for multiobjective optimization are based on the non-dominated evolutionary or genetic algorithms (Deb, 2001; Madeira et al., 2006). While many of the evolutionary algorithms such as genetic algorithm, cuckoo search algorithm, particle swarm optimization algorithm, artificial bee colony algorithm, harmony search algorithm and artificial immune algorithm are capable of generating Pareto-optimal topologies, a common drawback is that they require numerous **FEA**-runs, and therefore, from a practical point of view, their computational cost discourages their use to problems of large-scale topology optimization.

4. PROPOSED METHOD OF A HYBRID EVOLUTION STRATEGY AND NUMERICAL EXPERIMENTS

Optimization methods can be arranged into two groups: direct methods (represented, for instance, by optimization techniques that mimic the process of natural selection) and gradient-based methods. In direct search methods, only the objective function and the constraint values are used to conduct the search method, whereas gradient-based methods (e.g., the method of steepest descent) consist in computing the steepest-descent direction associated with the gradient, and then the optimal step-size along this direction. Compared to direct methods gradient-based methods faster converge near an optimal solution, but are not efficient in non-differentiable or discontinuous problems. One of the essential drawbacks of the steepest descent algorithm for problems of topology optimization is that it requires an initial guess. Based on local information arisen from the initial guess, the algorithm suggests a search direction. A unidirectional search is then performed along the search direction to find better solution. This better solution becomes the new solution and the above procedure is continued for a number of times. Figure 1 illustrates this procedure.

In addition, local search technique of gradient-based methods can easily be trapped in local minima. Hence, there is no guarantee that a steepest descent algorithm finds an optimal solution. In general, one can only say that it terminates in a critical point.

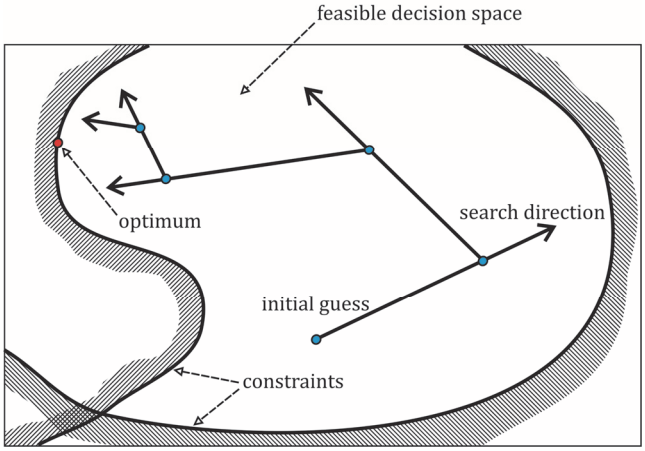


Fig. 1. *Path of optimum design.*

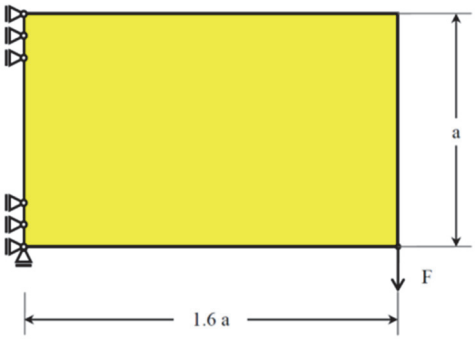


Fig. 2. *The design domain, boundary conditions and external load for the optimization of a cantilever beam.*

In this research, to overcome the above major problems in the gradient-based optimization methods a hybrid evolutionary-gradient algorithm (**HEGA**) is proposed to solve global numerical problems of topology optimization with continuous variables. The **HEGA** combines the multi-objective optimization technique of an evolution strategy based on ideas of adaptation and evolution with the gradient-based algorithm for solving the discrete problem of topology optimization (10). The hybrid evolution strategy has a powerful global exploration capability and uses natural problem-dependent representations, and primarily mutation and selection, as search operators to find the optimum-design domain Ω for the effective gradient-based algorithm, which searches one of local optimal topologies. In

common with evolutionary algorithms, the operators of the **HEGA** are applied in a loop. An iteration of the loop is called a generation. The sequence of generations is continued until a termination criterion is met.

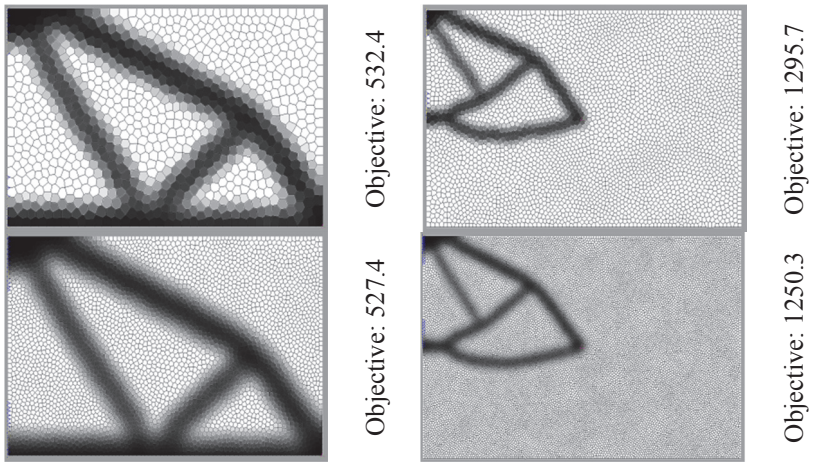


Fig. 3. *Impact of changes of the design domain and number of finite elements on the optimal topologies and corresponding compliances for* $f_V = 0.25$ *(the* f_V *is calculated for the design domain from figure 2).*

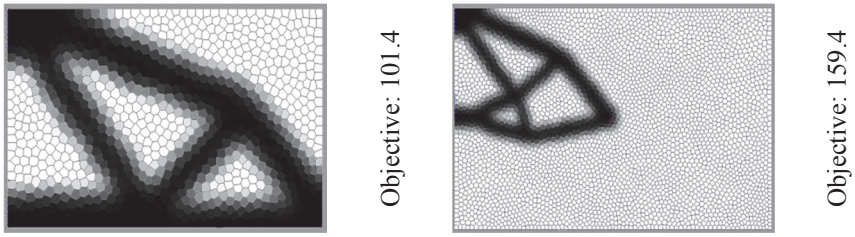


Fig. 4. *Impact of changes of the design domain on the optimal topologies and corresponding compliances for* $f_V = 0.4238$ *(the* f_V *is calculated for the design domain from figure 2).*

Assuming that initial population has N different individuals, which consist of finite sets of n points generated randomly, implicit descriptions of domain geometries can be provided by calculating the convex hull areas. In the plane the convex hull of a finite set $\mathbb{P}$ of points is the unique convex polygon, whose vertices are points from $\mathbb{P}$ and that contains all points of $\mathbb{P}$. According to an alternative definition of the convex hull, n points, which form the convex hull are compared to nails sticking out of the plane. Thus, the convex hull of $\mathbb{P}$ is an area enclosed by an elastic rubber band (tightened around all nails), which snaps minimizing its length. As far as real-valued search spaces are concerned, mutation is normally performed by adding a normally distributed random value to each vector component of point coordinates. The step size or mutation strength (i.e. the standard deviation of the normal distribution) is often governed by self-adaptation (see evolution window). Only fixed points, which are the support points do not change their coordinates in the numerical exploration of the evolution strategy.

The underlying principle behind the class of evolution strategy methods is as follows. First, a population of design domains Ω_s ($\forall s = 1, 2, \ldots, N$) implicitly defined by means of the convex hulls is generated. The concept of Voronoi diagrams plays a central role in meshing of the convex hulls. The used meshing scheme of a centroidal Voronoi tessellation is similar to the proposed by Vasconcellos and Maliska (2004). Second, the gradient-based algorithm searches local optimal topologies for each design domains Ω_s. Third, through stochastic methods, typically via mutation and selection some of the design domains Ω_s are modified, and also non-Pareto design domains are eliminated. Then, the cycle is repeated.

The MATLAB codes written by Talischi et al. (2012a, 2012b) were as a starting point for the development of computer software for the **HEGA**. The aim of carried out numerical experiments is to demonstrate the validity of the proposed method, as well as the necessity of its use. As an example, the problem outlined in figure 2 is considered. The calculation results are summarized in figures 3-5. For the calculations there are two different values of the prescribed volume fraction f_V = 0.25 and 0.4238.

5. CONCLUSIONS

Topology optimization has been implemented through the use of the hybrid evolutionary-gradient algorithm. The proposed method is a compromise between the computational speed of the effective gradient-based optimization methods and overall accuracy of evolution strategy calculations to determine the global optimum. The calculation results shown in figures 3 and 4 indicate that the choice of the design domain in the task of topology optimization has a significant impact on designs of compliant structures. However, if changes of the design domain are very limited as indicated in figure 5, they will have no significant effect on the compliance

outcome. It should be stressed that the **HEGA** is an universal method for topology optimization, which is susceptible to modifications due to the specific design tasks.

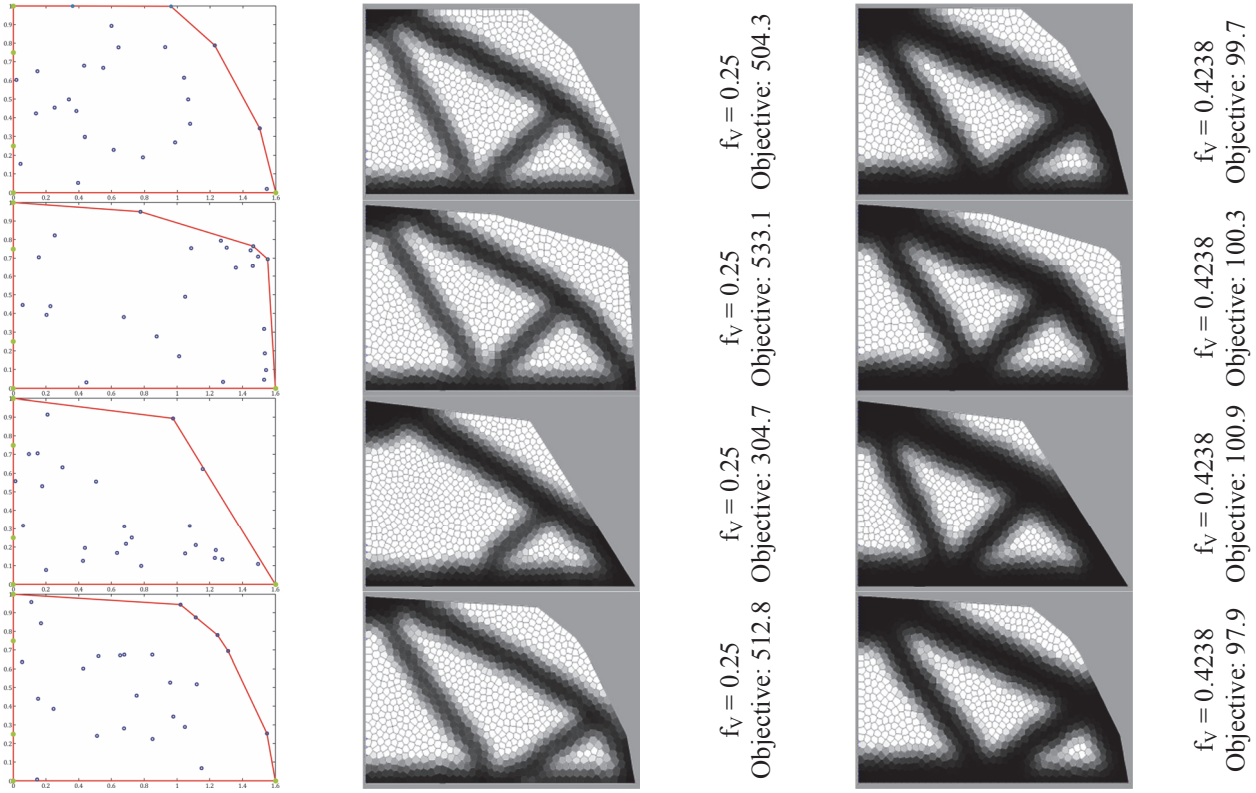


Fig. 5. *Generated design domains in the form of convex hulls together with corresponding to them optimal topologies and compliances for* $f_V = 0.25$ *and* $f_V = 0.4238$ *(obtained by means of the gradient-based algorithm).*

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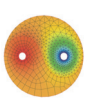

OPTYMALIZACJA PRZESTRZENNYCH STRUKTUR NOWOCZESNYCH MATERIAŁÓW ZA POMOCĄ HYBRYDOWEJ STRATEGII EWOLUCYJNEJ

Streszczenie

W niniejszej pracy zaproponowano nowy, hybrydowy algorytm gradientowo-ewolucyjny do numerycznego rozwiązywania zagadnień z zakresu globalnej optymalizacji topologii. Proponowany algorytm stanowi kompromis między bezpośrednimi i gradientowymi metodami optymalizacji. Krótka analiza porównawcza wyników obliczeń wykazuje walidację proponowanej metody, a także konieczność jej stosowania.

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