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THE RESOURCES OCCUPANCY IN RESOURCE – CONSTRAINED MULTI PROJECT SCHEDULING PROBLEM

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Abstract

This paper presents an adaptation of the Theory of Constraint (TOC) to the multi – project scheduling problem. Variables customer expectations have forced changes in the company management. Now more than 25% of economic activity suited to manage the projects. This concerns mainly the areas such as engineering, public works sector, aerospace and defense, shipbuilding, organizational consulting, etc. The project is a single, unique event, finished in a certain period of time without exceeding the established budget.The interpretation of TOC and constrained – based scheduling is a solution to maximize the number of project, which the enterprise is able to implement simultaneously. Scheduling problems arise in situations where a set of activities has to be processed by a limited number of resources during a limited period of time. The scheduling problem consists of resources allocation and resources scheduling - ordering of activities on each resource. The alternative way of resources occupancy is presented.

Key words: theory of constraint (TOC), multi - project scheduling, resource - constrained project scheduling problem, resources occupancy

1. INTRODUCTION

Globalization, increased competition, uncertainty of market behavior, fast-growing technology has a significant impact on the company's management. Market and customer requirements are the cause of a large diversity of products and customize them, which is particularly noticeable in small and medium-sized enterprises (SMEs). Most companies in the SME sector is target on producing variable assortment or produce a unique type of project. Consumers today are increasingly changing goods, not only because of that they have become useless. In a short time the customer changes the consumer goods, even if they do not lose their functions. Changes forced on consumers' attitudes that led to the change in the production process. For this reason, production orders are always treated as a new and unique, and how management is similar to the case of project management.

A constrained project is a set of activities, which must be performed according to some precedence and resource constraints to create unique product or service.

There are difficulties completing project on time, within budget and with full content. Problems in one project cascade into problems in other projects. One, big complex project is similar to multi – project environment in which few project should be executed simultaneously and share the same resources.

This paper presents an adaptation of the Theory of Constraint (TOC) to the multi – project scheduling problem. The Theory of Constraints is a management philosophy developed by Eliyahu M. Goldratt in the early 1980s. It is a system approach

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based on the assumption that every organization has at least one constraint. TOC uses the global safety time to schedule the project and stresses that the system has to have a constraint. Adaptation of this method for the project management is critical chain approach.

In the multi-project situation CC scheduling should be extended to drum buffer scheduling, which is a constraint of the multi-project environment and which limits a greater number of projects. The approach to project management known as Critical Chain Project Management (CCPM) provides mechanisms to allow a "whole system" view for projects. It avoids major impact of Parkinson's Law "Work expands to fill (and often exceed) the time allowed" at the task level while accounting for Murphy's Law "Whatever can go wrong, it will" at the project level. The scheduling mechanisms provided by CCPM require the elimination of task due dates from project plans. One benefit is that it allows those who use it to avoid the significant impact of "student syndrome" i.e., putting off work or expanding work to fill the time allowed. The application of the TOC assumes that almost all activities in a project can be reduced by up to 50%, but the safety time buffer called the Project Buffer (PB) has to be added at the end of the whole project. The PB should be equal to 50% of the project realization time. It gives the effect of reducing the duration of the project in Rand (2000).

This paper is organized in a following way: in Section 2 the main assumption of Theory of Constraints and its application for the resources – constrained project scheduling problem is presented. Section 3 describes problem formulation and resources availability in drum schedule. Section 4 contains example, and conclusions are described in section 5.

2. THE THEORY OF CONSTRAINTS

The Theory of Constraints, proposed by Eliyahu M. Goldratt is the approach used to develop different techniques of project management. Due to the TOC philosophy each system has at least one constraint that is the element which both limits and synchronizes production flow. This philosophy is based on five steps (identify the systems constraint, exploit the constraint, subordinate everything to the constraint, elevate the constraint, go back to the first step).

TOC uses the global safety time to schedule the project, and stresses that a system must have a constraint. The system constraint is that part of the system that constrains the objective of the system. This different kind of scheduling method requires the use of the three TOC improvement questions: what to change? to what to change it for? and how to cause the change? Goldratt (2000).

2.1. Critical Chain Project Management

A project is a set of activities which must be performed according to some precedence constraints requiring that some activities cannot start before some others have been completed. When resources constraints are not taken into account, a project can be represented by an acyclic directed graph. The popular project scheduling methods Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) are applied only for single project management. Generally CPM and PERT allow the minimization of project completion time, or the minimization of project completion cost through crashing, or shortening particular activities. About 90 % of projects are realized in the multi – project environment. Scheduling problems arise in situations in which a set of activities has to be processed by a limited number of resources during a limited period of time. The scheduling problem concerns of resources allocation and resources scheduling - ordering of activities on each resource. In multi – project environment, or complex project a problem of shared resources appears, and in this case the popular method like CPM or PERT cannot be used. Critical path in such problem should be extended to the Critical Chain. The TOC assumes that the critical path should be extended to activities using constrained resources. The Critical Chain (CC) is the method proposed for multi – project scheduling. The aim is to focus on critical areas, by identifying the CC, and to insert time buffers at the appropriate places (moments) in the project network developed by Goldratt, Rand (2000) .

First of all the company resource which is constraint should be identified. This resource becomes the drum for scheduling multiple projects. The drum sets the beat for the entire factory, for all company projects. In multi – project environment resources have to be divided among few projects – in such situation the problem of multitasking arises. In the project system the drum schedule determines the sequencing of projects. Multitasking delays the ability of successors tasks to start. The second step in CC is exploiting the resource by synchronizing the projects using that resource as a drum. Project synchronization eliminates some, if not all, resources overlapping. Bad multitasking is eliminated through giving the priority to projects and tasks for individual resources, particularly the capacity constrained resource on the basis of research performed by Rand, Leach, Goldratt (2000).

Benefits obtained from CCPM are presented in table 1.

The TOC method applied to the project management leads to 5 steps basis of research performed by Goldratt, Leach (2000).

1. Identification of the critical chain – identification of the system constraint
2. Determination of the shortest possible deadline for completion of the project, taking into account estimates of task execution time at 50% probability of timely delivery and the introduction of Project Buffer PB.
3. Introduction of additional feeding buffers.
4. Observation and control over the size of time buffers and utilizations; possible correction action.
5. Identification of the new system constraint – go back to step 1.

3. PROBLEM FORMULATION

In general, scheduling problems are NP-hard, there are no known algorithms to finde optimal solutions in polynomial time. In most general form the resource – constrained scheduling problem (RCSP) is defined as follows:

Given is:

- a set of executed activities,
- a set of renewable (e.g. labor) or non – renewable resources (e.g. raw materials, capital),
- a set of constraints which must be satisfied,
- a set of objectives.

Activities have duration, cost and resources capacities. Activities may be preemptive or non – preemptive (preemption means that some activities can be interrupted). Constraints define a feasibility of a schedule. According to Merkle D., Middendorf M., Schmeck (2002) and Wall (1996) objectives define the optimality of a schedule. Objective should be satisfied, constraints must be satisfied.

3.1. Research review

The research literature for the RCPSP is quite large. A great number of exact methods to solve the RCPSP are proposed in the literature. The exact methods applied to the RCPSP can be classified into three categories: dynamic programming, zero-one programming and implicit enumeration with branch and bound. Blazewicz, Lenstra and Rinnooy (1983) showed that the RCPSP as a generalization of the classical job shop scheduling problem belongs to the class of NP-hard optimization problems. Therefore, the use of heuristic solution procedures when solving large problem is well - founded. Most of the heuristics methods used for solving resource-constrained project scheduling problems either belong to the class of priority rule based methods or to the class of metaheuristic based approaches (Kolisch and Hartmann 1999). While exact solution methods are able to solve smaller problems, heuristic and metaheuristic approaches are needed for larger problem instances. Many metaheuristic methods, such as genetic algorithms (GA), simulated annealing (SA), tabu search (TS), and ant colonies (AC), have been applied to solve the RCPSP. Metaheuristics based on GA are the most common, e.g. Lee and Kim in [16], Alcaraz and Maroto in (2001). Simulated annealing algorithms in resource constrained project scheduling problem are presented by Boctor in (1996), Bouleimen and Lecocq in (2003). Tabu search based on metaheuristics are proposed by e.g. Merkle, Middendorf and Schmeck in (2002) proposed an ant colony approach to the RCPSP. Pritsker et.al (1969) proposed a zero – one programming approach for the multi – project scheduling. Most of the heuristic method used for solving RCMPSP belong to the class of priority rule based methods. Several approaches of this class have been proposed in the literature, e.g. Tsubakitani and Deckro (1990), Lawrence and Morton (1993), Wiley, Deckro and Jackson in (1998), Lova, Maroto and Tormos (2000).

3.2. Project scheduling and multi – project scheduling

The resource – constrained project scheduling problem (RCPSP) and resource – constrained multi – project scheduling problem (RCMPSP) can be characterized by the objective function, resources requirements, precedence relations and preemptive conditions. RCPSP involves assigning jobs to a resources (or a set of resources) with limited capacity in order to meet some predefine objectives. RCPSP is a problem of determining starting times of each activity of a project satisfying precedence and resource constraints in order to minimize the total project duration. Due to precedence constraints an activity can start only after the completion of all its predecessor activities. The execution of an activity cannot be interrupted and requires, for each period of its duration, constant amount of a subset of renewable resources. Many different objectives are possible and these depend on the goals of the decision maker. The most common objective is to find the minimum makespan (i.e. minimization of project duration). In the RCPSP, each activity has a single execution mode: both the activity duration and its requirements for a set of resources are assumed to be fixed, and only one execution mode is available for single activity. RCMPSP is a generalization of the RCPSP. In a RCMPSP environment a company has several concurrent projects, each consisting of a set of activities (each activity has several associated attributes) and each project has a corresponding precedence network. Projects depend on a common set of resources and are therefore related by resource constraints.

RCMPSP maps activities to resources in order to meet predefined problem. This problem can be characterized as follows: given is a set of activities $\mathbf{J} = \{1,\ldots,j,\ldots,n\}$ which should be realized on constrained resources $\mathbf{R} = \{1,\ldots,i,\ldots,r\}$, where set of capacity resources are $\forall i \in [1,r]$, $i > 0$. Each project activity $j \in J$ characterized by its duration time p_j, $(p_i,\ldots,p_j,\ldots,p_n)$ and resources requirements for i-th resource during scheduling activity j, which is given in matrix $\mathbf{BT}_{rn}$ – busy time (1).

$$\mathbf{BT}_{rn} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rn} \end{bmatrix} \quad (1)$$

The project schedule is represented by a vector of starting time $(s_1,\ldots,s_j,\ldots,s_n)$, where s_j is a starting time of activity j. If s_j is defined as a starting time of activity j, then the f_j is the finishing time of j activity. The completion time is represented by $\{s_n + p_n\} \mid n \in \mathbf{J}\}$

3.3. Definitions of resources

In the multi – project environment, according to some elements of the theory of constraints a key element is the resource that limits the number of projects (critical resource – drum) which one organization can handle. The drum is the critical resource which is constrained in project realization. Due to availability of resources we can distinguish different type of resources, like renewable (e.g. manpower, machinery) and nonrenewable (e.g. budget or materials). In this case only the renewable resources are taken into account. The project imposes mutual exclusion mode and rendezvous mode. Project activities are preemptive and described by resource occupancy. TOC leads to an understanding that all resources that the constraint must have excess capacity. Upstream of the constraint resource must have excess capacity that the constraint resources is never starved for work. Resources downstream of the constraint must have more capacity that the constraint to deal with fluctuations.

The occupancy of the constraint resource is described by a set of vector. The size of the vector is equal to the time to use and each vector element correspond to the one unit time. When the drum is free the element of the vector is equal 0 and when the resource is occupied in this unit time the element vector reflects the activity which is used. The definition of resources is based on Skołud (2000), Skołud and Zientek (2003).

Occupancy vector of the i-th resources fir the implementation of j-th activity.

$\mathbf{V}$ is a set of occupancy vector used in system

$$\mathbf{V} = [V_1, V_2, \ldots, V_i] \quad (2)$$

Each element of this set is vector V_i describing the state of *i-th* resource.

$$V_i = [o_{i1}, o_{i2}, \ldots, o_{ik}, \ldots, o_{it}] \quad (3)$$

o_{ik} - element of the vector V_i describing the state of the i-th resource in unit time k

$$o_{ik} = \begin{cases} 0 & \text{when the } i-\text{th resource is idle} \\ j & \text{when the resource isoccupied by the } j-\text{th activity} \end{cases} \quad (4)$$

Definition of the drum resource is given by the 4 conditions.

<u>Condition 1</u>

The finishing time of the activity is given by

$$f_j = s_j + p_j \quad (5)$$

Condition 2

There exists a time interval in the considered resource in which the resource is idle.

$$\forall r \epsilon R_i \exists \, k \epsilon V_i, \; V_i \epsilon \mathbf{V}, \; o_{ik} = 0 \qquad (6)$$

k – unit time,
o_{ik} – availability of considered resource.

Conditon 3

Considered time interval in C2 is the interval that is equal to the duration of the operations of the project to complete on that resource

$$\exists R_i, \; \forall t \epsilon \langle s_{j+1}, ..., s_{j+1} + \Delta o_{ik} \rangle, \; o_{ik} = 0 \qquad (7)$$

where Δo_{ik} is time interval of the idle i-th resource

Condition 4

Routing matrix is a matrix M of dimensions RxJ where R is the number of resources in system and J is the number of activities to project completion

$$M = RxJ \qquad (8)$$

Element of this matrix is:

$$m_{ij} = \begin{cases} p_{ij} & \text{duration of the activity } j \\ 0 & \text{when } j-th \text{ activity doesn't run through theresource} \end{cases} \qquad (9)$$

Problem

Renewable resources that are overloaded limit the number of projects that one organization can execute. The objective is to maximize the number of projects, which the company is able to implement simultaneously. In such problems relation type is finish-to-start, one project (or one unit of project) must end so that another could start.

The problem is how to described resource, which is constraint, to check the possibility of implementing new project? How to minimize makespan by exploit system resources to be effective for multi – project management?

4. EXAMPLE

There is a list of prioritized projects and a drum schedule in company. In multi – project environment a new project arrives. The problem is to fit (if you can) the new project into the system. The only way to schedule a new project is through the drum schedule. The new project may be of the lowest priority or it may receive higher than some of the ongoing projects.

Let assume an 3 resources which are the drum in organization. In this case drum is obtained from 5 steps of TOC theory. Three projects were accepted for realization (figure 1) which cannot be changed. The new project (4) is waiting for execution. In this case only renewable resources are considered.

The Gannt`s chart illustrated resources occupancy by the projects already implemented and new project according to conditions given in previous section. The occupancy vectors of the drum are:

V_3 = [0,1,1,1,0,0,0,1,1,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0]
V_2 = [0,0,2,2,2,2,2,2,2,2,2,0,0,2,2,2,0,0,0,0,0,0,0,0,0]
V_1 = [0,0,0,0,3,3,3,3,3,3,3,3,3,3,3,3,3,3,0,0,0,0,0,0,0]

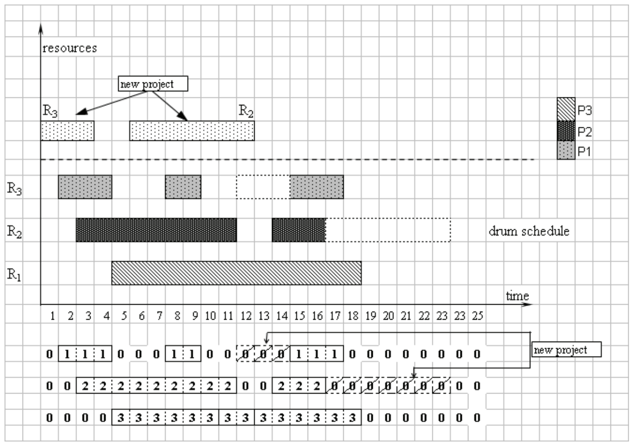

Fig. 1. *The drum schedule and the new project.*

The main objective of the method for presentation an occupancy resources is to check the possibility of implementing a new project based on the comparison of vectors in the range of occupancy resources designated by the CC's new design. Occupancy of each resource should be considered only within the limits of the vector Vi, in the moment at which projects are located.

Drum schedule with new project implemented:

V_3 = [0,1,1,1,0,0,0,1,1,0,0,4,4,4,1,1,1,0,0,0,0,0,0,0,0]
V_2 = [0,0,2,2,2,2,2,2,2,2,2,0,0,2,2,2,4,4,4,4,4,4,4,0,0]
V_1 = [0,0,0,0,3,3,3,3,3,3,3,3,3,3,3,3,3,3,0,0,0,0,0,0,0]

In this example four conditions are satisfied. From second condition we can see that R2 and R3 are available. From third condition we see that time interval of the idle i-th resource is enough for implementation of new project. The last one condition is fulfilled because the precedence constraints are satisfied.

If the new project (5) arrives it should be introduced to the timing of the drum on the assumption that it will start as soon as possible, due to the CC project. However, it should be planted over the timetable of the drum. All projects on the priority index lower than the priority of the new project should be moved above the drum schedule. Then again sets a timetable for all projects starting from project to project of higher priority.

Main purpose of this problem is to check the possibility of a new project implementation. If the conditions 2 and 3 are satisfied the acceptance of the new project into the system is possible. From the Gantt chart we can see that second and third condition are satisfied. For further research an Swarm Algorithm approach is proposed to the RCPSC as a alternative way of scheduling. The concept of swarm intelligence originates from the observations of animals and insects in nature. Swarm algorithms have gained popularity recently because of their design and ability to solve hard problems. The metaheuristic based on Ant Colony Optimization (ACO), which belongs to swarm algorithms, is proposed as an alternative way of scheduling. Heuristic methods typically require less time and/or space than exact methods. Scheduling heuristics operate on a set of tasks and determine when each task should be executed. ACO is modeled on the foraging behavior or Argentine ants. Real ants are capable of finding shortest path from a food source to the nest. In ACO several generations of artificial ants search for good solution. Every ant builds a solution step by step going through several probabilistic decisions. If ant find a good solution mark their paths by putting some amount of pheromone (which is guided by some problem specific heuristic) on the edges of the path. The following ants are attracted by the pheromone so that they search in the solution space near previous good solutions (Dorigo et al., 1996; Merkle et al., 2002).

5. SUMMARY

In this paper the TOC philosophy is proposed to the multi – project environment. The alternative method of notation considering the resource occupancy was proposed The critical chain for a single project is usually not the constraint for an enterprise performing multiple projects. It is necessary to identify the multi – project constraint and go through the focusing steps to adapt the CCPM process to firms with multiple projects. The drum resources is the constraint for multi – project scheduling. The main step is to select the drum resource and subordinate all projects with access to it. Elimination of bad multitasking has the greatest impact on overall enterprise project throughput.

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ZAJĘTOŚĆ ZASOBÓW W HARMONOGRAMOWANIU PROJEKTÓW W ŚRODOWISKU WIELOPROJEKTOWYM

Streszczenie

W artykule zaprezentowano adaptację Teorii Ograniczeń w środowisku wieloprojektowym. Obecnie ponad 25% działalności gospodarczej nadaje się do zarządzania przez projekty (inżynieria, doradztwo, przemysł lotniczy itp.). Projekt to jednorazowe działanie niepowtarzalne, złożone, skończone w określonym czasie, które prowadzi do zrealizowania unikatowego zdarzenia. Zastosowanie teorii ograniczeń do harmonogramowania w warunkach ograniczeń zasobowym umożliwia maksymalizację projektów przyjętych do realizacji. Problemy harmonogramowania powstają w sytuacjach, gdzie zbiór czynności musi zostać zrealizowany na ograniczonej liczbie zasobów w zadanym czasie. W artykule zaproponowano alternatywne podejście do zajętości zasobów w harmonogramowaniu projektów w środowisku wieloprojektowym.

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