Publishing House
A K A P I T

COMPUTER METHODS IN MATERIALS SCIENCE
Informatyka w Technologii Materiałów

Vol. 11, 2011, No. 2

THE METHOD OF ACCELERATION OF SOLVING EQUATIONS SYSTEM IN CASE OF SHAPE AND/OR BOUNDARY CONDITIONS CHANGES

NORBERT SCZYGIOL, JULIUSZ MIKODA*

Institute of Computer and Information Sciences, Czestochowa University of Technology
ul. Dąbrowskiego 73, 42-200 Częstochowa, Poland
**Corresponding author: juliusz.mikoda@icis.pcz.pl*

Abstract

This paper presents new approach to the problem of conducting numerical simulations in case of applying shape and/or boundary conditions changes resulting from technological and/or constructional modifications. Presented approach allows to shorten the time required for solving linear equations system by using results obtained from simulation for original model (before modifications) to initialize vector of solution in the second turn of computation (for modified model). The conjugate gradient method was used to solve the system of equations. The paper presents results of simulations conducted with use of such approach. In the first case model of some constructional element was changed by adding a slot. In the second case, the modification involves changing value of temperature used in boundary conditions. The results obtained in both cases confirm that described method can decrease time required for solving linear system of equations. The costs of such approach are relatively small and does not affect total time of computation. Using the approach described in this paper, in connection with mesh reconstruction, can give significant acceleration. In the analysed cases the total acceleration resulting from the application of these two techniques was approximately 30%.

Key words: finite element method, conjugate gradient method, mesh reconstruction, solving linear systems of equations

1. INTRODUCTION

Advanced numerical methods constitute the basis of physical phenomena simulations. The Finite Element Method (FEM) (type of use - Sczygiol et al. 1997), one of the most popular advanced numerical method, attracts a lot of attention in the scientific community, though other methods, such as the Finite Difference Method (FDM), Boundary Element Method (BEM), Finite Volume Method (FVM), are also used. All of these methods required solving large systems of linear equations. Usually, solving such systems of equations with use of direct methods is very difficult or even impossible. Therefore, iterative methods are used. These methods use successive approximations to obtain more accurate solutions at each step of iteration (Barrett et. al. 1994). Although the non-stationary iterative methods, such as Conjugated Gradient Method, can be highly effective, solving such large systems of equations is still very time-consuming task.

Usually, the computations are conducted only once for a given task, but sometimes there is a need to implement slight changes in the geometry of the analysed domain (or domains) or some modifications of parameters used in boundary conditions. In such cases the finite elements mesh generation process and solving system of equations have to be conducted once again from the beginning. The mesh reconstruction method, described by Sczygiol & Mikoda (2008) can help decrease the amount of time

 ISSN 1641-8581

required for the first of these tasks, which is mesh generation.

In situation, when changes have to be applied to the model after the first turn of computations has been made, the process of solving system of equations can be also accelerated. It can be done by using results obtained from computations performed before applying changes, to initialize the vector of solution in first iteration of the second turn of computations.

2. INITIALIZATION OF SOLUTION VECTOR

The Conjugated Gradient Method is one of the most effective iterative method. Its name is derived from the fact, that this method generates sequences of orthogonal (conjugated) vectors during successive iterations. Each successive vector is closer to a solution.

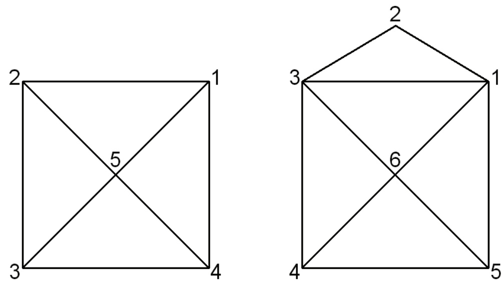
At the beginning, the solution vector has to be initialized with some values. When the computations are conducted for the first time, this vector is initialized with zero value. During the preparation of the next series of computations, results obtained from the previous one are known. When the modifications of the model are slight it can be assumed that the differences between solutions will also be small. This observation can be used to accelerate the process of solving system of equations during the second turn of computations. New solution vector can be initialized with values from previous solution. In such case, the initial solutions vector and resulted solution vector are close and the convergence of iterative process is faster.

3. RENUMBERING OF NODES

In case, when only the parameters used in boundary conditions are changed and the shape of the considered model remains unchanged, the numbers of nodes do not have to be renumbered. The values from solution obtained for unchanged model can be directly moved to initial solution vector build for the second turn of computations.

In the other case, when the shape of model has been changed, the finite element mesh have to be reconstructed. During this operation new nodes of finite element mesh can be inserted into the existing mesh and some nodes of the existing mesh can be removed. This requires renumbering of the nodes that belong to new finite elements mesh. Because the numbers of nodes have been changed, the values of solution vector obtained for unchanged model can not be directly moved to new initial vector of solution.

During finite elements mesh reconstruction after applying shape changes, conducted with use of method described by Sczygiol & Mikoda (2006), table of nodes renumbering is created. This table consists of pairs of node numbers. The exemplar table of nodes renumbering is presented in figure 1.



1	1
3	2
4	3
5	4
6	5

Fig. 1. *Exemplar table of nodes renumbering.*

The second column of this table consists of node numbers before mesh reconstruction. The numbers in the first column are numbers of new nodes corresponding to the old indexes of nodes in the second column. For example, the new node 1 corresponds to the old node with the same number. New node with number 2 does not correspond to any node from first mesh. New node with index 3 corresponds to old node with number 2.

This information are used to move values of the solution vector obtained for unchanged shape to the new initial vector. The first value from this vector should be placed at the first position in new vector, the second one on the third place and so on.

4. EXPERIMENTAL VERIFICATION OF PROPOSED METHOD

To verify the efficiency of the method described above, a series of simulations were performed. The simulations were performed for a steady heat transfer phenomena in a model of some constructional element. The simulations were conducted for the three dimensional model and four sizes of mesh elements. Required accuracy was set to 1e-3 and 1e-6. The Dirichlet boundary conditions were imposed on all model boundaries. The temperature on one surface (marked on the figure 2a) was set to T1 = 850 K and on the others to T2 = 350 K. Figure 2a shows the analysed model.

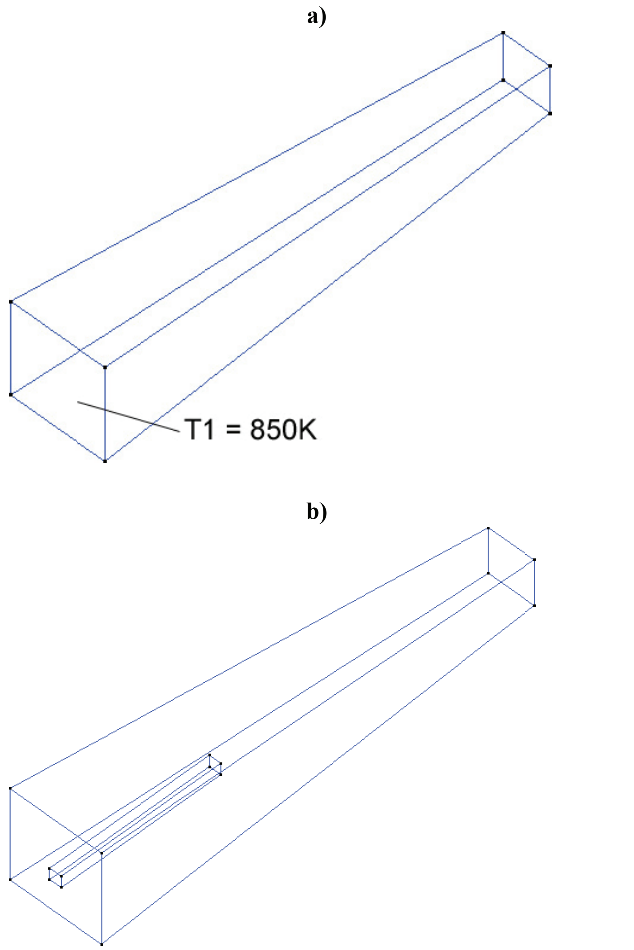


Fig. 2. Model of the constructional element a) original model b) after applying constructional changes.

Table 1. Simulation for accuracy 1e-3 (changing shape of model).

Size of elements [m]	Mesh generation time [s]	Mesh reconstruction time [s]	Solving system of equations				Acceleration	
			Init. vec. filled up with zero values		Init. vec. filled up with values from previous simulation			
			i_num_{zero} [-]	t_{zero} [s]	$i\_num_{non\text{-}zero}$ [-]	$t_{non\text{-}zero}$ [s]	a_{iter} [%]	a_{time} [%]
0.025	8.82	2.17	107	9.93	98	9.7	8.41	2.32
0.020	16.87	3.99	206	35.55	196	34.17	4.85	3.88
0.015	42.88	9.67	187	81.05	165	72.79	11.76	10.19
0.012	84.2	18.55	241	195.38	215	176.67	10.79	9.58
0.010	150.32	32.55	274	385.96	245	349.57	10.58	9.43

After the first simulation, constructional changes were applied to the model. The shape was changed by adding a slot. The modified model of this element is presented in figure 2b. The Dirichlet boundary conditions, with temperature set to T1 = 850 K, were imposed to the new surfaces created inside this slot.

New finite elements mesh was generated by using mesh reconstruction. The information obtained from this process, along with results of the first simulation were used to initialize vector of solution for the second simulation with non-zero values. The numbers of iterations required to obtain requested accuracy and times of these operations are shown in table 1 and table 2. The last two columns present the accelerations a_{time} and a_{iter} determined from formulas 1 and 2:

$$a_{time} = \frac{t_{zero} - t_{non-zero}}{t_{zero}} 100\% \qquad (1)$$

where:
t_{zero} – time of solution – initial vector filled up with zero values,
$t_{non\text{-}zero}$ – time of solution – initial vector filled up values from previous simulation.

$$a_{iter} = \frac{i\_num_{zero} - i\_num_{non-zero}}{i\_num_{zero}} 100\% \qquad (2)$$

where:
i_num_{zero} – number of iteration – initial vector filled up with zero values,
$i\_num_{non\text{-}zero}$ – number of iteration – initial vector filled up values from previous simulation.

Conducted simulations show that approach described in this paper can reduce number of iteration required to obtain requested accuracy by even 12%. Reduction of number of iterations can results in shortening time of solving system of equations by about 10%.

The only costs of this approach are times of creating table of nodes and moving values to initial vector. Such table is being created during renumbering of nodes, which is part of mesh reconstruction process, so it does not require any additional time. The computational complexity of moving values to initial vector is linear and it does not affect the time of simulation appreciably.

Table 2. *Simulation for accuracy 1e-6 (changing shape of model).*

Size of elements [m]	Mesh generation time [s]	Mesh reconstruction time [s]	Solving system of equations				Acceleration	
			Init. vec. filled up with zero values		Init. vec. filled up with values from previous simulation			
			i_num_{zero} [-]	t_{zero} [s]	$i\_num_{non-zero}$ [-]	$t_{non-zero}$ [s]	a_{iter} [%]	a_{time} [%]
0.025	8.82	2.17	154	14.28	147	13.81	4.55	3.29
0.020	16.87	3.99	272	46.11	260	44.39	4.41	3.73
0.015	42.88	9.67	271	114.59	250	106.72	7.75	6.87
0.012	84.2	18.55	342	272.41	315	253.02	7.89	7.12
0.010	150.32	32.55	399	552.45	363	506.65	9.02	8.29

Table 3. *Simulation for accuracy 1e-3 (changing the temperature).*

Temperature T1 on boundaries [K]	Solving system of equations				Acceleration	
	Init. vec. filled up with zero values		Init. vec. filled up with values from previous simulation			
	i_num_{zero} [-]	t_{zero} [s]	$i\_num_{non-zero}$ [-]	$t_{non-zero}$ [s]	a_{iter} [%]	a_{time} [%]
800	274	356.57	209	292.08	23.72	18.09
820	274	356.57	195	267.95	28.83	24.85
840	274	356.57	175	247.97	36.13	30.46
860	274	356.57	175	248.35	36.13	30.35
880	274	356.57	195	274.07	28.83	23.14
900	274	356.57	209	291.96	23.72	18.12

Table 4. *Simulation for accuracy 1e-6 (changing the temperature).*

Temperature T1 on boundaries [K]	Solving system of equations				Acceleration	
	Init. vec. filled up with zero values		Init. vec. filled up with values from previous simulation			
	i_num_{zero} [-]	t_{zero} [s]	$i\_num_{non-zero}$ [-]	$t_{non-zero}$ [s]	a_{iter} [%]	a_{time} [%]
800	399	517.86	329	447.13	17.54	13.66
820	399	517.86	321	436.91	19.55	15.63
840	399	517.86	301	411.01	24.56	20.63
860	399	517.86	301	411.69	24.56	20.50
880	399	517.86	321	437.01	19.55	15.61
900	399	517.86	329	447.73	17.54	13.54

The second simulation shows a situation where the shape of the model has not been changed, but the parameters used in boundary conditions has been changed. During the first simulation the temperature on boundaries (T1) was set to 850 K. To verify usefulness of the described method this temperature was changed. The computations were made for the biggest mesh (size of element 0,01 m) and six different values of temperature T1. Tables 3 and 4 present results obtained for requested accuracy 10e-3 and 10e-6.

The results show that approach described in this paper can be successfully used in case of changing boundary conditions. The time acceleration reached in conducted simulations was in the range of 10 to 30 percent and the acceleration counted on the basis of iterations number was even higher.

5. SUMMARY

In the case when some constructional or technological modifications have to be implemented to analyzed model, the finite elements mesh has to be generated again for modified area's shape and the computations have to be performed once again. If changes are not significant, the mesh reconstruction can reduce time required for this process but it can also give additional possibility of acceleration. The auxiliary data, that are created during mesh reconstruction, can be used to speed up the most time-consuming part of numerical simulations, which is solving system of equations. Using the approach described in this paper in connection with mesh reconstruction, can give significant acceleration. In the analysed cases the total acceleration resulting from the application of these two techniques together (mesh reconstruction and initialization of solution vector with non-zero values) was approximately 30%. Conducted research shows also that the same approach can be used in case of boundary condition changes.

REFERENCES

Barrett, R., Berry, M., Chan, T.F., Demmel, J., Donato, J.M., Dongarra, J., Eijkhout, V., Pozo, R., Romine, C., Van Der Vorst, H., 1994, *Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods*, SIAM.

Sczygiol, N., Mikoda, J., 2008, Renewed Generation of Finite Elements Meshes in Parts of Domain Changing from Structural and Technological Respects, *Proc. RELMAS'2008*, *Assessment of Reliability of Materials and Structures: Problems and Solutions*, 6, 322-325.

Sczygiol, N., Mikoda, J., 2006, Niestrukturalny generator przestrzennych siatek elementów skończonych oparty na metodzie postępującego frontu, *Proc. Conf. KomPlasTech 2006*, eds, Szeliga, D., Pietrzyk, M., Kusiak, J., Szczawnica, 61–68, (in Polish).

Sczygiol, N., Szwarc, G., Olas, T., Nagórka, A., 1997, Object-oriented analysis of solidification modeled by finite elements (in Polish), *Solidification of Metals and Alloys*, 30, 233-242.

METODA PRZYSPIESZENIA PROCESU ROZWIĄZYWANIA UKŁADU RÓWNAŃ W SYTUACJI ZMIANY KSZTAŁTU I/LUB WARUNKÓW BRZEGOWYCH

Streszczenie

W artykule zaprezentowano nowe podejście do problemu przeprowadzania symulacji numerycznych w sytuacji, gdy do modelu wprowadzane są modyfikacje kształtu i / lub warunków brzegowych wynikające ze zmian konstrukcyjnych i / lub technologicznych. Zaprezentowane podejście pozwala na skrócenie czasu potrzebnego na rozwiązanie układu równań liniowych poprzez wykorzystanie wyników symulacji dla oryginalnego modelu (przed modyfikacją) do zainicjowania wektora rozwiązań w drugiej turze obliczeń (dla zmodyfikowanego kształtu). Do rozwiązania układu równań zastosowana została metoda sprzężonych gradientów. W artykule zaprezentowano wyniki symulacji przeprowadzonych z w wykorzystaniem opisanej metody. W pierwszym przypadku model został zmodyfikowany poprzez dodanie wąskiego otworu. W drugim przypadku modyfikacje polegały na zmianie temperatury wykorzystanej w warunkach brzegowych. W obu przypadkach uzyskane wyniki potwierdziły, że opisana metoda pozwala skrócić czas potrzebny na rozwiązanie układu równań liniowych. Koszt związany z zastosowaniem takiego podejścia jest relatywnie mały i nie wpływa na całkowity czas wykonania obliczeń. Zaprezentowana w pracy metoda, w połączeniu z rekonstrukcją siatki może pozwolić na uzyskanie znaczącego przyspieszenia. W analizowanych przypadkach łączne przyspieszenie wynikające z zastosowania tych technik wyniosło około 30%.

Received: November 9, 2010
Received in a revised form: November 24, 2010
Accepted: November 24, 2010