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APPLICATION OF THE FRACTIONAL OSCILLATOR EQUATION TO SIMULATIONS OF GRANULAR FLOW IN A SILO

TOMASZ BLASZCZYK[2*], EWA KOTELA[1], JACEK LESZCZYNSKI[1]

[1] *Czestochowa University of Technology, Department of Heating, Ventilation and Air Protection*
ul. Dabrowskiego 73, 42-200 Czestochowa, Poland
[2] *Czestochowa University of Technology, Institute of Mathematics*
ul. Dabrowskiego 73, 42-200 Czestochowa, Poland
**Corresponding author: tomblaszczyk@gmail.com*

Abstract

In this paper we consider a problem of continuous operation of silos. We have focused our attention on the two extreme experimental cases. The first one is considered as a mass flow and the second one is known as a blockade of particles in a silo. In our present work we have analysed in more detail the outflow of granular material in the intermediate regimes between the above mentioned extreme cases. Considering the transition between stable and unstable operation we proposed a novel mathematical model of the silo emptying. This model involves a differential equation with fractional derivatives.

Key words: arching, fractional differential equations, silo emptying, funnel flow, DEM

1. INTRODUCTION

Behavior of granular materials during the silo emptying is an interesting issue, considered both by experimental and theoretical methods (Balevicius et al., 2007; Corwin et al., 2005; Niedostatkiewicz & Tejchman, 2003; Schulze, 2007). During the emptying of the silo granular material can flow generally in two modes, mass flow of material or funnel flow (Schulze, 2007). During the mass outflow one can observe the linear loss of mass over time. Mass flow is only possible, if the hopper walls are sufficiently steep and/or smooth, and the bulk solid is discharged across the whole outlet opening. If a hopper wall is too flat or too rough, funnel flow will appear. In this case one can observe the nonlinear loss of mass outflow over time. In case of funnel flow only the bulk solid which is placed in the area slightly above the outlet is in motion first. The bulk solid adjacent to the hopper walls remains at rest and is called the “dead” zone. This bulk solid can be discharged only when the silo is emptied completely.

Improper silo operation may lead to blockage of outflow from the silo. Therefore, it becomes an important issue to work out such a method that would allow us to predict at which time there may be blockage of the silo.

In the literature (Arvalo et al., 2007; Artoni et al., 2009; Balevicius et al., 2007; Kozicki & Tejchman, 2001; Kozicki & Tejchman, 2005) one can find a lot of studies in which the authors propose different approaches to the mathematical description of the dynamics of the outflow of granular material from the silo. They have certain limitations especially when we consider granular cohesion dynamics.

In this paper we propose a new mathematical model used to describe the outflow of granular mate-

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rial from the silo. This model is based on fractional calculus. An ordinary differential equation containing fractional derivatives is proposed. The equation is called the fractional oscillator equation.

2. EXPERIMENTAL SETUP

We used the experimental setup described by Blaszczyk (2010). The experiment consisted of filling the container (figure 1) with granular matertial and subsequent process of emptying the silo recorded with a digital camera.

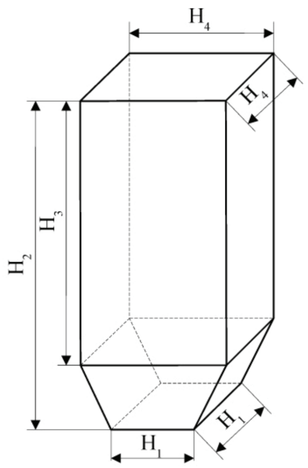


Fig. 1. Silo dimensions: H_1 = 0.0425 m, H_2 = 0.399 m, H_3 = 0.299 m, H_4= 0.073 m.

The experiment was performed for two cases: dry pea (figure 2a) and wet pea with capilary cohesion c^{max} = 10,6 kPa (figure 2b).

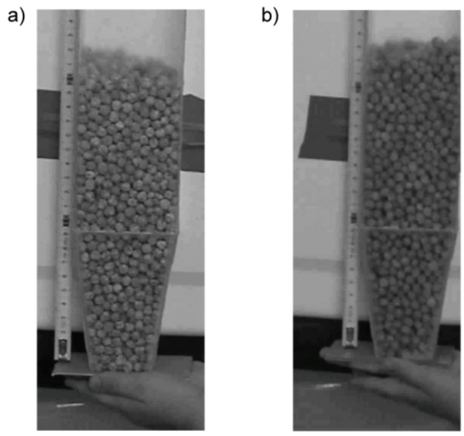


Fig. 2. Two initial states of granular material filling the silo: a) dry pea, h_{bed} = 0.21 m, b) wet pea, h_{bed} = 0.23 m.

When the granular material was dry (figure 2a), there was a mass outflow and the total duration time of emptying the silo was T_d = 0.88±04s. In the second case shown in figure. 2b, when the granular material was wet, there was a blockade of particles in a silo. Outflow of particles has been stopped. They have formed an arch at the bottom of the silo hopper. The blockade of the outflow occurred as a result of capillary action of strong liquid bridges between contacting particles and between particles and the silo wall.

The outflow of granular material in the states which are intermediate between the above mentioned two extreme cases will be the object of a more detailed analysis. Appropriate experimental studies of these phenomena would be technically complicated and would require an advanced research equipment. Therefore we decided to carry out simulations of intermediate states of the outflow of granular material from the silo using the DEM (Leszczynski, 2004).

3. FRACTIONAL OSCILLATOR EQUATION

Considering the transition between stable and unstable operation we propose a novel mathematical model of the silo emptying. This model involves a fractional differential oscillator equation in the form:

$$ {}^{C}D^{\alpha}_{0+}D^{\alpha}_{T-}h(t)+\lambda^{\alpha}h(t)=0, \quad t\in[0,T] \qquad (1)$$

with conditions:

$$h(0)=h_{bed}\,[m] \quad h(T)=0\,[m] \qquad (2)$$

where $h(t)$ is the height of the granular bed during emptying, h_{bed} is the initial bed height, and D^{α}_{T-} is the right Riemann-Liouville fractional derivative which is defined as (Kilbas et al, 2006):

$$D^{\alpha}_{T-}h(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_t^T\frac{h(s)}{(s-t)^{\alpha}}ds \qquad (3)$$

and ${}^{C}D^{\alpha}_{0+}$ represents the left Caputo fractional derivative (Kilbas et al, 2006):

$${}^{C}D^{\alpha}_{0+}h(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{h'(s)}{(t-s)^{\alpha}}ds \qquad (4)$$

Solution of Eqn.(1) has a complex form (Klimek, 2008), represented by series of fractional integrals or G-Meijer functions (Kilbas, 2006) and it is difficult to apply, for example to do a chart. Therefore, in the paper (Blaszczyk, 2010) the following numerical scheme to solve equation (1) has been proposed. Let $t \in [0,T]$ then the interval is divided into N parts, so that $0=t_0<t_1<...<t_{N-1}<t_N=T$, where it is assumed constant time step $\Delta t = t_{k+1} - t_k$, for k = 1,

…, N-1. Let us denote $h(t_k) = h_k$, then for k = 1, …, N-1 we obtain a discrete form of equation (1) as

$$\Delta t^{-2\alpha}\sum_{j=0}^{k}\left[v(k,j)\sum_{i=j}^{N}w(j,i)h_i\right]+\lambda^\alpha h_k=0 \quad (5)$$

where

$$v(k,j)=\frac{1}{\Gamma(2-\alpha)}\begin{cases}-k^{1-\alpha}+(k-1)^{1-\alpha} & for\ j=0\\ (k-j+1)^{1-\alpha}-2(k-j)^{1-\alpha}+(k-j-1)^{1-\alpha} & for\ j=1,\dots,k-1\\ 1 & for\ j=k\end{cases} \quad (6)$$

and

$$w(k,i)=\frac{1}{\Gamma(2-\alpha)}\begin{cases}(N-k-1)^{1-\alpha}-(N-k)^{1-\alpha} & for\ i=N\\ (i+1-k)^{1-\alpha}-2(i-k)^{1-\alpha}+(i-k-1)^{1-\alpha} & for\ i=k+1,\dots,N-1\\ 1 & for\ i=k\end{cases} \quad (7)$$

Having Eqn. (1) and its solution in the discrete form we can simulations granular flow in a silo.

4. RESULTS

Given the experimental results and the results obtained with the discrete element method, emptying a silo was simulated with a fractional oscillator equation. We analyzed the change of the height of the granular bed placed in a silo with respect to time depending on the size of capillary cohezion c.

Figure 3 presents changes of the height of granular material over non-dimensional time during the emptying of the silo for all cases determined by the capillary force c. We assumed that $t^* = t/T$ indicates the non-dimensional time $t^* \in [0,1]$ and T is the total emptying time $T \in \{0,88;25,34;31,31\}$.

Analyzing the change of the height of the granular layer over the time we can say that in figure 3a a mass outflow is observed. Almost linear relationship exists between the time and the height. The increase in the value of capillary force, which can be seen in figures 3b and 3c, causes irregular outflow in the granular bed. For the maximum values of capillary forces we observed blocking the outflow of granular material (figure 3d). Additionally in all cases we can observe good agreement between the experimental data, the DEM simulations and the solution of the fractional oscillator equation (1).

Analyzing the behavior of parameters α and λ we can observe the same relationship between these parameters and capillary cohesion. When α tends to 1 and λ tends to 0, we observe a mass outflow. If the parameter α decreases and λ increases then we observe irregularities in the operation of the silo. On the contrary, when α and λ reach values of 0.58 and 2.1, respectively, we can say that the outflow of particles from a silo has been blocked.

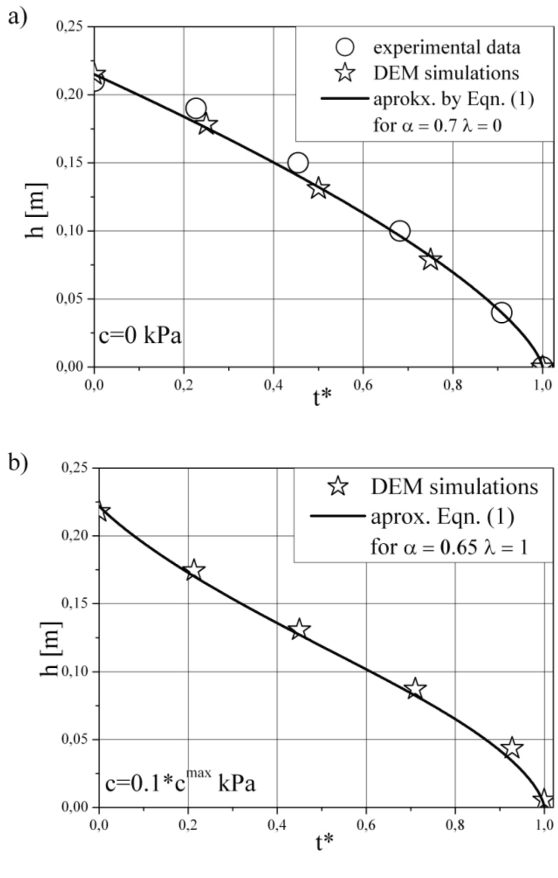
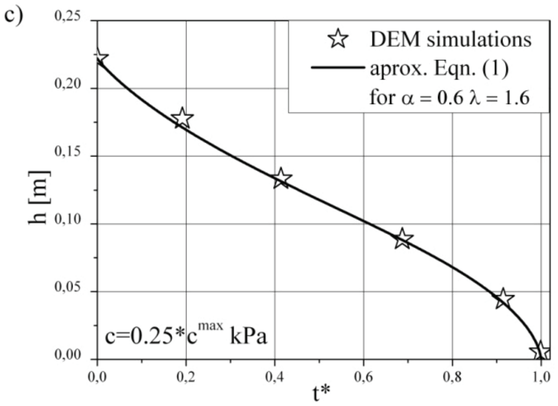


Fig. 3a, b. *Changes the height of granular material over non-dimensional time during the emptying of the silo:*
(a) the mass flow for c = 0 kPa,
(b) the irregular flow for $c = 0.1 \cdot c^{max}$ *kPa,*



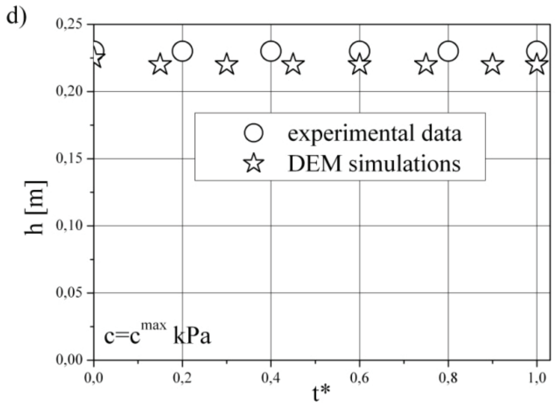


Fig. 3c, d. *Changes the height of granular material over non-dimensional time during the emptying of the silo:*
(c) the irregular flow for $c = 0.25 \cdot c^{max}$ *kPa,*
(d) the silo blocade for $c = c^{max}$ *kPa.*

5. CONCLUSIONS

Our studies show that the model based on fractional oscillator equation, describes well the change of the height of the granular bed in time during the emptying of the silo. Thus, in industrial conditions it is sufficient to observe the increase in time of the mass of granular material which flows from the silo – not necessarily until the silo is completely empty. Then, using the fractional model given by equation (1), we can approximate the measurement data. On the basis of values for α and λ we can predict how close to the blockade of the outflow we are.

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ZASTOSOWANIE RÓWNANIA FRAKCYJNEGO OSCYLATORA DO SYMULOWANIA WYPŁYWU GRANULATU Z SILOSU

Streszczenie

W pracy rozpatruje się problem stabilnej pracy silosu. Eksperymentalnie rozważa się dwa skrajne przypadki. W pierwszym analizuje się wypływ masowy granulatu, natomiast w drugim rozważa się blokadę zbiornika. W celu bardziej szczegółowej analizy wypływu granulatu z silosu rozpatruje się dodatkowo stany pośrednie pomiędzy wypływem masowym a blokadą zbiornika. W oparciu o analizę przejścia pomiędzy stabilną i niestabilną pracą proponuje się nowy matematyczny model opróżniania silosu. Model ten zawiera równanie różniczkowe z pochodnymi niecałkowitego rzędu.

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