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APPLICATION OF GENETIC ALGORITHM FOR DAMAGE IDENTIFICATION OF NON-HOMOGENOUS TAPERED BEAM

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Abstract

Non-homogeneous structural members such as beams are very important in various engineering applications and for experimental analysis purposes. A minor damage on any part of the structure reduces the strength of the structure and leads to a major failure. The analysis of non-homogenous beam becomes complicated due to the change of material properties from point to point. However, it becomes much more complicated when there exists a taper on such type of beams. In this paper, a new formulation of an objective function for the genetic search optimization procedure along with the residual force method is presented for the identification of macroscopic structural damage in a non-homogeneous tapered beam. The developed model requires experimentally determined data as input and detects the location and extent of the damage in the beam. Here, numerically simulated data using finite element models of structures are used to identify the damage at a reasonable level of accuracy. Damage parameters given theoretically are compared by the present procedure and are found to be in good agreement.

Key words: damage factor, non-homogeneous material, genetic algorithm (GA), residual force, eigen value, tapered beam

1. INTRODUCTION

During the last three decades, vibration based methods have been developed and applied to detect structural damage in the civil, mechanical and aerospace engineering communities (Cawley & Adams, 1979; Stubbs & Osegueda, 1990). These methods are based on the fact that the vibration characteristics of structures (namely frequencies, mode shapes, and modal damping) are functions of the structural physical parameters such as mass, stiffness and damping. Structural damage usually causes a decrease in structural stiffness, which produces changes in the vibration characteristics of the structure. Gawronski and Sawicki (2000) employed modal norms to determine damage locations. The residual force concept has received wide attention for application to damage detection and assessment. Residual force provides an objective function to be minimized for achieving the dynamic balance.

Identifying the structural damage with the measured vibration data is an inverse approach in mathematics. The usual damage detection methods minimise an objective function, which is defined in terms of the discrepancies between the vibration data identified by modal testing and those computed from the analytical model (Hajela & Soeiro, 1990; Chakraverty et al., 2006). However, these conventional optimization methods are gradient based and usually lead to a local minimum only. A global optimization technique is needed to derive a more accurate and reliable solution.

In the last two decades, since first introduced by Holland (1975), Genetic Algorithm (GA) has been widely applied to various optimization problems (Goldberg, 1989). Many authors have recently taken

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up this optimization problem using neural networks (Tsou & Shen, 1994; Barai & Pandey, 1995), Genetic Algorithm (Mares & Surace, 1996; Luh & Wu, 1999) and neural network with GA (Marawala & Chakraverty, 2006) by studying the variation of localized damage as a function of modal test data and machine learning. As compared with the traditional optimization and search algorithms, GA search from a population of points in the region of the whole solution space, rather than a single point, and can obtain the global optimum. Moreover, GA has the advantage of easy computer implementation. These properties make GA successful and powerful in the field of structural optimization (Rajasekaran & Pai, 2003).

All the work of damage identification mentioned above is done for structures with uniform thickness. It is also found that a lot of work has been done on vibration analysis of beams and plates with variable thickness (Subramanian & Raman, 1996; Al-Kaabi & Aksu, 1990; Ashour, 2001; Singh & Chakraverty, 1994). But to the best of our knowledge investigation on damage identification of tapered structure is scarce. It may be due to the fact that the inclusion of the tapered function makes the governing equation complex and thereby the damage detection also becomes complicated by traditional methods. Moreover as structural members are frequently operated in extremely thermal and mechanical environment, various kinds of new materials have been developed. Very really, new materials viz. functional inhomogeneous materials have received attention and practical applications over a multi-scale range from aerospace field to MEMS one. Research and development of functional inhomogeneous materials such as functionally graded materials and composite materials contribute to weight saving and improvement in stiffness or material strength in members. Such weight saving in members often results in considerable decrease in thickness and stiffness and flexible members cause vibration resulting from time variation of heating, which is called thermally induced vibration. In this respect, Redecop (2006) has studied the free vibration characteristics of the non-homogenous shells. Bhangale and Ganesan (2006) have analysed the buckling and vibration behaviour of functionally graded sandwich beam. When thin-walled members are subjected to cyclic thermal and mechanical loads, there exists a lot of chance of failure of the member. Methods to identify damage in a structure in particular to homogeneous beams, plates and shells are already developed. But to the best of our knowledge damage identification method for non-homogeneous structure is not done. Again, non-homogeneity function creates more complexity in damage identification. The case of tapered non-homogenous beam, makes the governing equation much more complicated. Accordingly, a powerful and reliable method such as Genetic Algorithm (GA) has been utilized here which may identify and quantify the damage.

An important class of damage identification methods is based on the updating or modification of structural matrices. Accordingly, investigations are done using the residual force vector in damage detection problems using optimal matrix modification. Residual force provides an objective function to be minimized for achieving the dynamic balance. Chen and Garba (1988) put forward a theory for assessing the occurrence, location and extent of potential damage using on-orbit response measurements. This method detects damages by using the minimum norm solution of the residual force equation. Chiang and Lai (1999) presented a two-stage structural damage detection method. The residual force vector is also used to localize damage preliminarily and the simulated evolution method is employed to determine damage extent. Three techniques for damage quantification are studied by Yang and Liu (2007) to obtain damage extents after the suspected damaged elements are determined, the first is the algebraic solution of the residual force equation, minimum-rank elemental update (MREU) technique is the second and the third happens to be the natural frequency sensitivity method. These damage detection methods are demonstrated on a numerical example with the measurement noises.

This paper introduces the concept of residual force vector to specify an objective function for an optimization procedure, which is then solved using a Genetic Algorithm. Rao et al. (2004) have used this procedure for homogenous uniform cantilever beam, truss structures and portal frames. Panigrahi et al. (2007) addressed the problem of damage identification in a uniform cantilever beam of homogenous material only by changing the selection methods in GA. In (Panigrahi et al., 2008), the authors have analysed damage detection of tapered beam by taking five elements into consideration. Here tapered beams of non-homogenous material has been considered. Non-homogeneity parameters are introduced in the governing equation to develop the model. Damage parameters as used corresponds to the reduction in stiffness of an element from which the structure is composed of. GA is employed to determine the values of these parameters by following an iterative process. When the objective function is optimized, values of the parameters indicate the state of the structure. Experimental data were simulated numerically by using finite element model of non-homogenous beam with and without noise and simulations have been done. It is seen that the identified damage factors are in good agreement with the theoretical one. Computer programme using MatLab is employed to find the location and extent of the damage.

As regards, in this paper, the section 5 of the present paper is divided into five parts. In the first part, damage assessment of uniform beam with homogenous material is discussed. In the second part, uniform beam with non-homogenous material is considered. Damage assessment of taper beam of homogenous material and tapered beam with non-homogenous material are considered in third and forth part respectively. In the last part, a special case is considered by putting slope parameter and non-homogeneity factor as zero in the governing equation for tapered beam of non-homogenous material and results are compared with that of uniform beam of homogenous material.

2. RESIDUAL FORCE METHOD

This section describes the construction of dynamics of damaged structures. The governing equation of motion of the dynamics of a multi degree freedom system is given by

$$[M]\{\ddot{X}(t)\} + [K]\{X(t)\} = F(t) \qquad (1)$$

where *[M]* and *[K]* are *(n X n)* system mass and stiffness matrices and *X (t)* and *F (t)* are *(n X 1)* physical displacement and applied force vectors.

The j^{th} eigen value equation for ambient vibration associated with equation (1) is

$$[K]\{\phi_j\} - \lambda_j\,[M]\{\phi_j\} = 0 \qquad (2)$$

where λ_j and ϕ_j are the j^{th} eigen value and corresponding eigen vector.

In the finite element model of the structure, the global stiffness can be represented as a sum of the expanded element stiffness matrices.

$$[K] = \sum_{i=1}^{m} [k]_i \qquad (3)$$

where k_i represents the expanded stiffness matrix of the i^{th} element and *m* is the total number of elements.

When damage occurs in a structure, the stiffness matrix of the damaged structure $[K_d]$ can be expressed as a sum of element stiffness matrices multiplied by damage factors associated with each of the *m* elements α_i (i = 1,2,…m), resulting from the damage.

Then, stiffness matrix of damaged structure may be given by

$$[K_d] = \sum_{i=1}^{m} \alpha_i [k]_i \quad where\ \alpha_i \in [0,1] \qquad (4)$$
$$and\ \ m = Number\ of\ elements$$

The values of the parameters fall in the range 0 to 1. The value α_i =1 indicates that the element is undamaged and α_i=0 or less than 1 implies completely or partially damaged element respectively.

If it is assumed that the experimental natural frequencies and mode shapes of the damaged structure continue to satisfy the eigen value equation (2), the j^{th} mode of the damaged structure can be written as

$$[K_d]\{\phi_{jd}\} - \lambda_{jd}\,[M]\{\phi_{jd}\} = 0 \qquad (5)$$

where λ_{jd} is the experimentally determined eigen value corresponding to the j^{th} mode shape of the damaged structure. Furthermore as already pointed out, the stiffness matrix is directly affected by the damage and the mass matrix *M* is assumed to be unaltered.

By substituting equation (4) in equation (5), an expression for residual force vector for j^{th} mode in terms of α_i can be written approximately (Mares & Surace, 1996 ; Rao et al., 2004) as

$$R_j = -\lambda_{jd}\,[M]\{\phi_{jd}\} + \sum_{i=1}^{m} \alpha_i [k]_i\,\{\phi_{jd}\} \qquad (6)$$

where R_j is a column matrix of (*nX1*) order for j^{th} mode. However, the overall residual matrix *R* will be of (*nXn*) order as all the modes have been considered here. *R* will be zero, only if correct sets of α_i are introduced under available damaged modal information λ_{jd} and ϕ_{jd}.

3. IMPLEMENTATION OF GENETIC ALGORITHM

GA is a search method based on Darwin's theory of evolution and survival of the fittest. Based on the concept of genetics, GA simulates the evolutionary process numerically. Analogous to genes in genetics, GA represents the parameters in a given problem by encoding them in a string. Instead of finding the optimum from a single point in traditional mathematical optimization methods, in GA, a set of points, that is, a population of coded strings, is used to search for the optimal solution. Simple GA consists of three operators: reproduction, crossover, and mutation (Goldberg, 1989; Michalewicz, 1994).

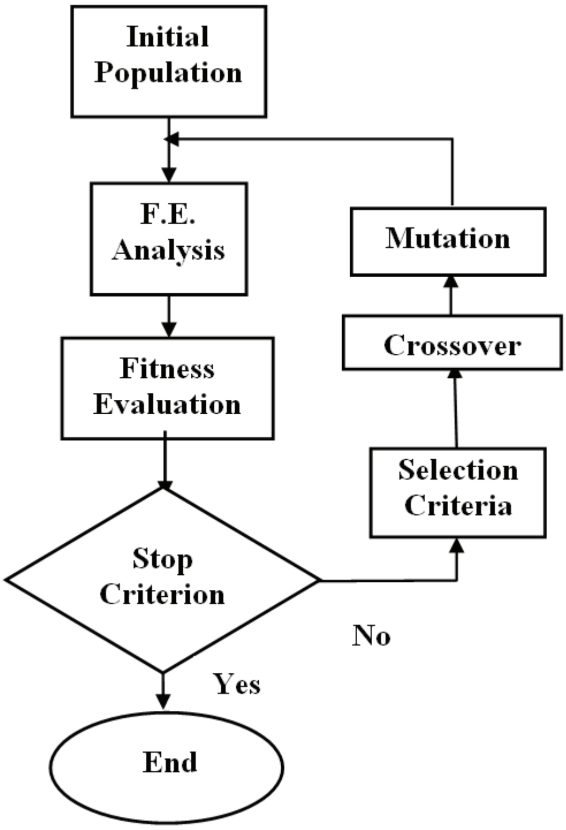

Fig. 1. Flow Chart of Genetic Search

To implement GA, it is necessary first to devise a general coding system for the representation of the design variables. Most commonly the design variables are coded by a bit-string. Next step of the procedure is reproduction, which incorporates the concept of natural selection. The fitness of different members of the population must be evaluated before mating to produce the next generation. There are a number of methods of mating pool selection out of which roulette wheel and Tournament selection are mostly used by number of authors for reproduction purpose. In this paper a method known as steady-state selection is selected for reproduction purpose. The main idea of the selection is that bigger part of the chromosome should survive to next generation. After the reproduction phase is over, the population is enriched with better individuals. Crossover operator is applied to the matching pool with a hope that it would create a better string. Following the crossover, the strings are subjected to mutation. The problem presented in the section below for which the search procedure adopted is illustrated by flow chart in figure 1. The procedure is repeated until the new generation ceases to improve the objective function that shows the occurrence of the convergence.

4. OBJECTIVE FUNCTION FOR GA FROM THE RESIDUAL FORCE VECTOR

From equation (6), it is found that the residual force vector is a *(n X n)* matrix where n is number of modes. If $[K_d]$ and $[M]$ are real symmetric matrices it can be shown that the diagonal terms of matrix *[R]* are zero, when a correct set of λ_d and ϕ_d are introduced. Hence the objective function of damage factors in the present situation is as follows:

$$f(\alpha_1, \alpha_2, \ldots\ldots \alpha_m) = \sqrt{R_{11}^2 + R_{22}^2 + \ldots\ldots + R_{nn}^2} \qquad (7)$$

where m is the number of elements and n is the number of modes.

$$V = {C_1}/\{C_2 + f(\alpha_1, \alpha_2, \ldots\ldots \alpha_m)\} \qquad (8)$$

Here our problem is to find out first the minimum residual forces. The fitness function V in the present task is an inverse function defined as below C_1 represents a constant used to control the value of the fitness function and C_2 represents a constant used to build a well defined function for the ideal case (Mares & Surace, 1996). The values of C_1 and C_2 are taken equal to 1 in this work (Rao et al.,2004). The genetic search procedure requires a proper selection of crossover and mutation operators. After some trials, the GA was set up as follows: Population Size -20, crossover probability-0.25 and mutation probability –0.01. Each structural parameter α_i was represented as a 10-bit binary numbers with variable limits 0 to 1.

5. ILLUSTRATIVE EXAMPLES

5.1. Uniform Cantilever Beam with Homogenous Material

A cantilever beam with homogenous material (figure 2) is considered first for the damage detection and extent of the damage using residual force vector method along with genetic algorithm. The beam is simulated numerically with a finite element model taking five elements. Each element is having both translation and rotational degrees of freedom at each nodal point to give a total of twelve. Because the fixed-point degree of freedom (dof) has zero rotational and translation movements, the total dof are ten. The properties of the beam chosen are as follows: modulus of Elasticity E = 70.3 GPa; Cross-sectional area A= 1.82 x 10^{-4} m^2; Moment of Inertia I= 1.46 x 10^{-9} m^4; density ρ = 2685 kg/m^3; Total length of the beam l = 0.5 m.

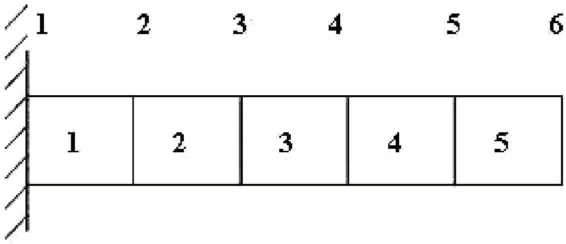

Fig. 2. *Cantilever Beam under Consideration*

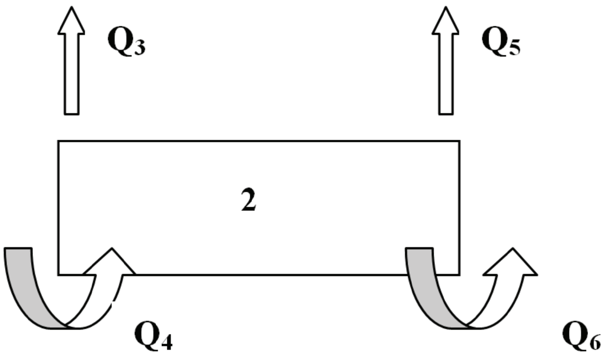

Fig. 3. *One Element Force Diagram*

Figure 3 shows the second element of the beam. Q_3 and Q_5 are the transverse displacement and Q_4 and Q_6 are the rotational vectors. There are total four degrees of freedom. For the element as shown l_e=0.1 m and other parameters remain same as of the full beam.

The element mass and element stiffness matrices for this element may be written as

$$m^e = \frac{\rho A_e l_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ & 4l_e^2 & 13l_e & -3l_e^2 \\ & & 156 & -22l_e \\ & & & 4l_e^2 \end{bmatrix} \qquad (9)$$

$$\text{and}\quad k^e = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ & 4l_e^2 & -6l_e & 2l_e^2 \\ & & 12 & -6l_e \\ & & & 4l_e^2 \end{bmatrix} \qquad (10)$$

Two different situations for this case are considered as (a) beam is in a state of undamaged and (b) the beam having element 2 damaged partially to an extent of 40% and element 5 by 30%. From equations 9 and 10, the global mass and stiffness matrices were calculated for undamaged beam. Again by reducing the stiffness of the second element by 40% and 5th element by 30%, the global stiffness matrices were calculated for the damaged structure.

Now FE analysis is performed to solve the eigen value problem of these two situations and the vibration frequencies are presented in table 1. It is found as usual that the frequencies in damaged structure in all modes are lower than the undamaged one.

The modal data from table 1 are employed as input to the model for finding out the values of damage factors from which the location and extent of damage may be identified. Figure 4 shows the best value of the fitness function verses the number of iterations. Here, the best value was established at iteration number 29 because when the iteration number was increased there was no improvement in the solution. Accordingly figure 4 shows this behaviour up to 500 iterations.

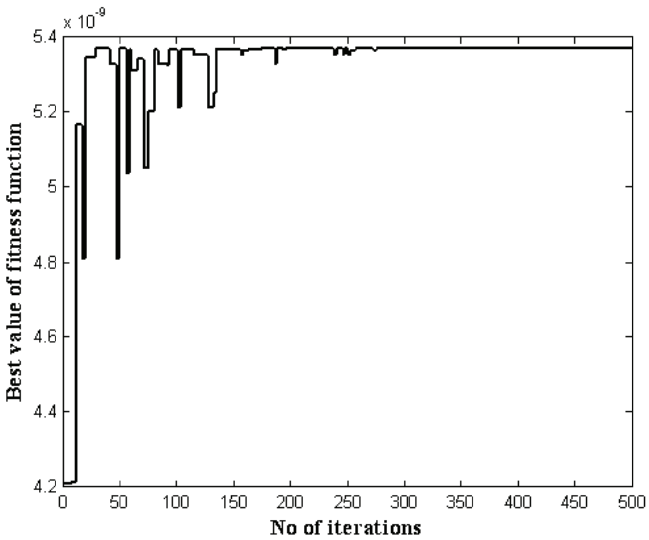

Fig. 4. *Graph for Maximum Value of Fitness Function Verses No. of Iterations*

Table 1. *Comparison of first six natural frequencies between undamaged and damaged cantilever with homogenous material*

	Frequency Parameter (rad/sec)					
Undamaged beam	203.83	1277.98	3589.46	7090.92	11769.16	19551.83
Damaged beam	179.26	1138.01	3020.38	6425.67	10480.85	17915.37

From table 2 it reveals that in both the damaged and undamaged situations the theoretical and GA identification results are in good agreement.

Table 2. Results of identified damage factors (α_i) of cantilever beam of homogenous material

Element No.	Undamaged Situation (a)		Damaged Situation (b)	
	Theoretical	Identified	Theoretical	Identified
1	1.0	0.95	1.0	0.94
2	1.0	0.96	0.6	0.57
3	1.0	0.91	1.0	0.94
4	1.0	0.93	1.0	0.97
5	1.0	0.95	0.7	0.68

5.2. Uniform Cantilever Beam with Non-homogenous Material

Here the same cantilever beam (figure 2) with non-homogenous material is considered. The modulus of elasticity varies along with the length of the beam. The modulus of elasticity at any distance x from free end is given by

$$E=E_0(1+0.5\ e) \qquad (11)$$

where E_0 = Modulus of elasticity at the free end and e = x / *l* and other parameters same as previous example. Figure 3 shows the second element of the tapered beam. Q_3 and Q_5 are the transverse displacement and Q_4 and Q_6 are the rotational vectors. There are total four degrees of freedom. For the element as shown l_e=0.1 m and other parameters remain same as that of the full beam.

Putting Eq. 11 in Eq. 9 and 10, m^e and k^e will both become the functions of the non-homogeneity parameters. By putting the appropriate non-homogeneity parameters, the values can be computed. Two different situations for this case are considered as (a) beam is in a state of undamaged, (b) the beam having element 2 damaged partially to an extent of 40% and element number 5 by 30%. From equations 9 and 10 global stiffness and mass matrices were calculated for case (a) i.e. the undamaged beam. Only global stiffness matrices were computed for case (b) and (c) i.e. damaged beams reducing the stiffness parameter.

Now FE analysis is performed to solve the eigen value problem of these two situations and it is found as usual that the frequencies in damaged structure in all modes are lower than the undamaged ones. These modal data are employed as input to the model for finding out the values of damage factors from which the location and extent of damage can be identified. Figure 5 shows the best value of the fitness function verses the number of iterations for above case (b).

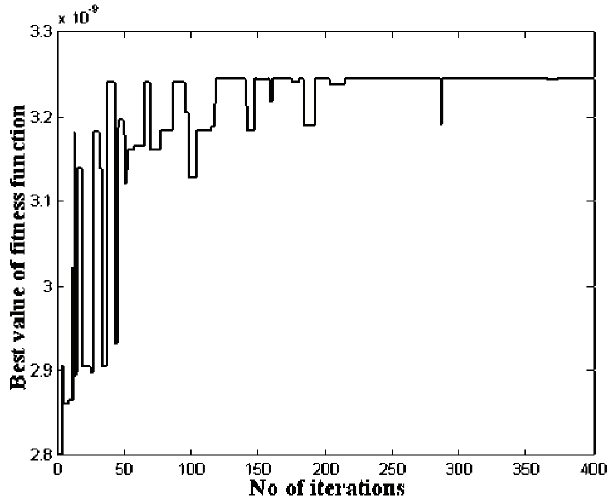

Fig. 5. Graph for Maximum Value of Fitness Function Verses No. of Iterations for Case (b)

As discussed previously, it may be seen from figure 5 that the best value is achieved after 80 iterations. Again it is worth mentioning that there is no further development of the best value after these iterations.

Table 3. Results of identified damage factors (α_i) of uniform cantilever beam of non-homogenous material

Element No	Undamaged Situation Case (a)		Damaged Situation Case (b)	
	Theoretical	Identified	Theoretical	Identified
1	1.0	0.95	1.0	0.97
2	1.0	0.96	0.6	0.57
3	1.0	0.95	1.0	0.94
4	1.0	0.96	1.0	0.96
5	1.0	0.94	0.7	0.66

From table 3, it may be seen that in both damaged and undamaged situations the theoretical and GA identification results are in good agreement for the present problem.

5.3. Tapered cantilever beam with homogenous material

Here a cantilever beam with variable thickness (figure 6) is considered. The properties of the beam are as follows Width= b= 0.0186m, the height at the free end= H_0 = 9.81 × 10^{-3} m, Taper parameter or the slope = $r= (H-H_0) / L = 0.006$ and other parameters same as previous example. Height at any point at a distance *l* from the free end of the beam is given by

$$h=H_0 + \{(H-H_0)\div L\}l=H_0+r\ l \qquad (12)$$

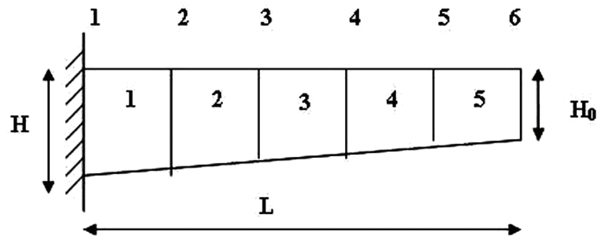

Fig. 6. Cantilever Beam under Consideration

By putting this h parameter in area and moment of inertia, these may be written as

$$Area = A_e = b\left[H_0 + \{(H - H_0) \div L\}l\right] \quad (13)$$

$$Moment\ of\ Inertia = I = \left[b\{H_0 + \{(H - H_0) \div L\}l\}^3\right] \div 12 \quad (14)$$

Figure 7 shows the third element of the tapered beam. Q_5 and Q_7 are the transverse displacement and Q_6 and Q_8 are the rotational vectors. There are total four degrees of freedom. For the element as shown l_e=0.1 m and other parameters remain same as of the full beam.

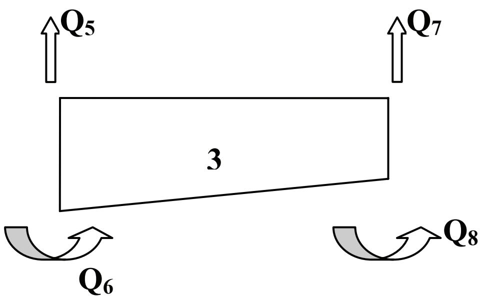

Fig. 7. One Tapered Element Force Diagram

Table 4. *Comparison of first six natural frequencies between undamaged Case (a) and damaged tapered cantilever Cases (b) and (c)*

	Frequency Parameter (rad/sec)					
Case (a)	179.17	1261.79	3660.76	7285.05	12133.40	20260.63
Case (b)	170.76	1123.43	3462.90	6737.11	11638.00	19419.26
Case (c)	150.18	1052.87	2919.11	6080.30	10007.20	18634.21

Table 5. *Results of identified damage factors (α_i) of tapered cantilever beam of homogeneous material*

Element No	Undamaged Situation Case (a)		Damaged Situation Case (b)		Damaged Situation Case (c)	
	Theoretical	Identified	Theoretical	Identified	Theoretical	Identified
1	1.0	0.95	1.0	0.96	1.0	0.95
2	1.0	0.97	1.0	0.94	0.6	0.56
3	1.0	0.98	0.7	0.65	1.0	0.94
4	1.0	0.92	1.0	0.96	1.0	0.93
5	1.0	0.95	1.0	0.97	0.7	0.67

Putting Eq. 13 and 14 in Eq. 9 and 10, m^e and k^e will both become the functions of the taper parameters. By putting the appropriate thickness parameter, the values can be computed.

Three different situations with r = 0.006 for this case are considered as (a) beam is in a state of undamaged, (b) the beam having element 3 damaged partially to an extent of 30% (c) the beam having element 2 damaged partially to an extent of 40% and element number 5 to an extent of 30%.

From Eq. 9 and 10, the global stiffness and mass matrices were calculated for undamaged beam putting original values of stiffness parameters for the case (a). For the case (b) the global stiffness matrices were calculated by reducing the stiffness value of the 3rd element by 30% and for the case (c) by reducing the stiffness value of the 2nd element by 40% and 5th element by 30%. The mass matrix will remain same as that of undamaged case. Now FE analysis is performed to solve the eigen value problem of these three situations and presented in table 4. It is found as usual that the frequencies in damaged structure in all modes are lower than the undamaged ones.

The modal data from table 4 are employed as input to the model for finding out the values of damage factors from which the location and extent of damage can be identified. Figures 8 and 9 show the best value of the fitness function verses the number of iterations for case (b) and (c) respectively.

It may be seen from figures 8 and 9 that the best value is achieved at 25 and 105 iterations respectively.

Table 5 demonstrates that the theoretical and GA identification results are in good agreement for both damaged and undamaged situations.

5.4. Tapered cantilever beam with non-homogenous material

In this section, the tapered cantilever beam (figure 6) with a different slope parameter along with non-homogenous material is considered. The modulus of elasticity is considered to vary along with the length of the beam. So, let us suppose that the modulus of elasticity at any distance x from free end is given by $E=E_0(1+0.5e)$ where E_0 = Modulus of elasticity at the free end and e = x / l and other parameters same as previous example.

Putting Eq. 11, 13 and 14 in Eq. 9 and 10, m^e and k^e will both become the functions of the taper and non-homogeneity parameters. By putting the appropriate thickness parameter, the values can be computed. Three different situations with r = 0.02 are considered as (a) beam is in a state of undamaged, (b) the beam having element 3 damaged partially to an extent of 30% (c) the beam having element 2 damaged partially to an extent of 40% and element number 5 to an extent of 30%.

This section investigates the problem with and without noise. Accordingly, the cases (a), (b) and (c) are studied first without noise. Then, for simulating to an experimental measurement, the natural frequencies and the mode shape are perturbed randomly as follows;

Noise level I.
Imposing noise of 1% in frequency and 2% in mode shape.

Noise level II
Imposing noise of 2% in frequency and 5% in mode shape.

From equations 9 and 10, the global stiffness and mass matrices of tapered non-homogenous beam were calculated in a similar manner as the previous case. FE analysis is performed to solve the eigen value problem of these three situations with different noise levels and is found as usual that the frequencies in damaged structure in all modes are lower than the undamaged ones.

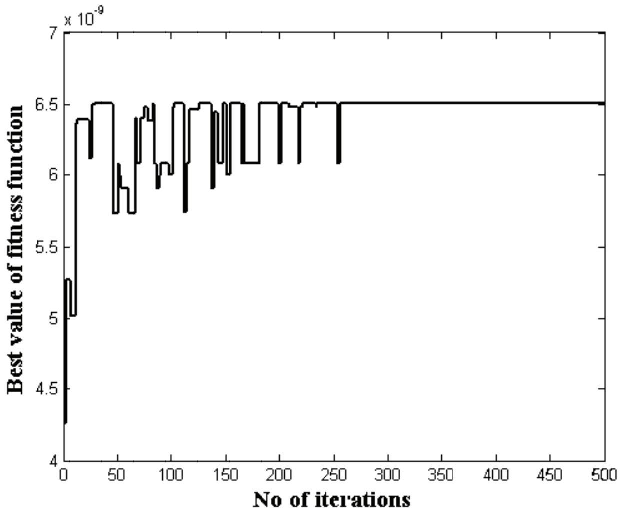

Fig. 8. Graph for Maximum Value of Fitness Function Verses No. of Iterations for Case (b)

The above data are employed as input to the model for finding out the values of damage factors from which the location and extent of damage can be identified. Figures 10 and 11 show the best value of the fitness function verses the number of iterations for case (b) with noise level I and case (c) with noise level II respectively.

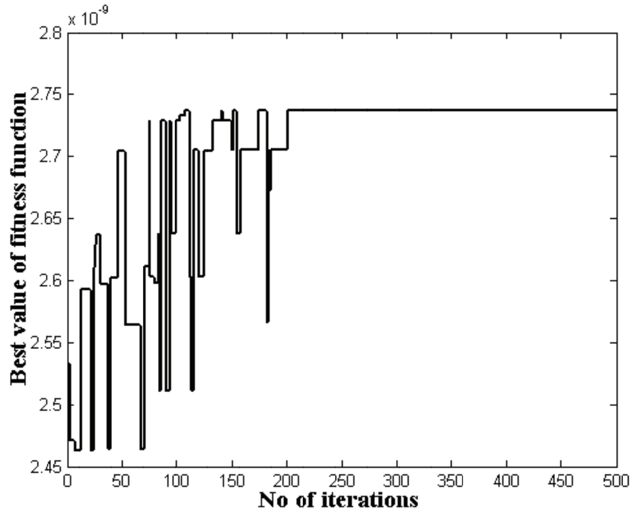

Fig. 9. Graph for Maximum Value of Fitness Function Verses No. of Iterations for Case (c)

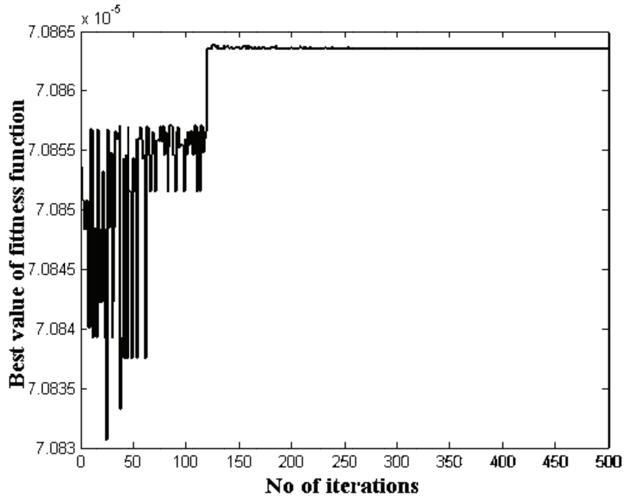

Fig. 10. Graph for Maximum Value of Fitness Function Verses No. of Iterations for Case (b) with noise level I

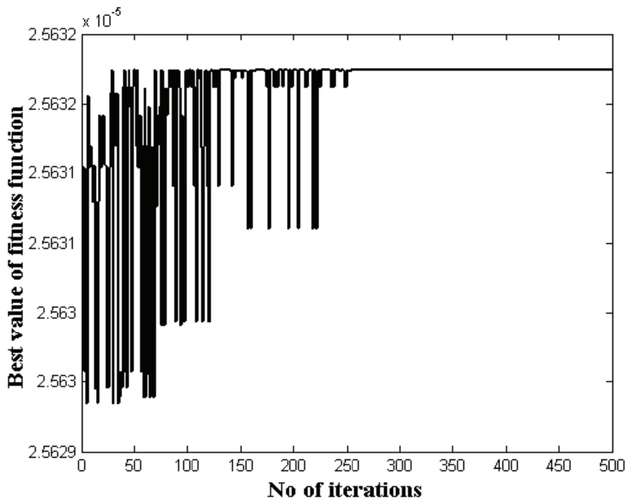

Fig. 11. Graph for Maximum Value of Fitness Function Verses No. of Iterations for Case (c) with noise level II

***Table 6.** Results of identified damage factors (α_i) of non-homogenous cantilever beam for Case (a)*

Element No	Theoretical damage factor	Without noise		Noise level I		Noise level II	
		Identified damage factor	% of error	Identified damage factor	% of error	Identified damage factor	% of error
1	1.0	0.96	4.0	0.95	5.0	0.90	10.0
2	1.0	0.95	5.0	0.93	7.0	0.92	8.0
3	1.0	0.98	2.0	0.95	5.0	0.91	9.0
4	1.0	0.92	8.0	0.94	6.0	0.89	11.0
5	1.0	0.91	9.0	0.90	10.0	0.92	8.0

***Table 7.** Results of identified damage factors (α_i) of non-homogenous cantilever beam for Case (b)*

Element No	Theoretical damage factor	Without noise		Noise level I		Noise level II	
		Identified damage factor	% of error	Identified damage factor	% of error	Identified damage factor	% of error
1	1.0	0.96	4.0	0.94	4.0	0.91	9.0
2	1.0	0.94	6.0	0.95	5.0	0.92	8.0
3	0.7	0.67	4..3	0.65	7.1	0.74	5.7
4	1.0	0.96	4.0	0.94	6.0	0.96	4.0
5	1.0	0.95	5.0	0.93	7.0	0.88	12.0

***Table 8.** Results of identified damage factors (α_i) of non-homogenous cantilever beam for Case (c)*

Element No	Theoretical damage factor	Without noise		Noise level I		Noise level II	
		Identified damage factor	% of error	Identified damage factor	% of error	Identified damage factor	% of error
1	1.0	0.93	7.0	0.95	5.0	0.92	8.0
2	0.6	0.55	9.1	0.54	10.0	0.66	11.7
3	1.0	0.95	5.0	0.91	9.0	0.93	7.0
4	1.0	0.95	5.0	0.92	8.0	0.87	13.0
5	0.7	0.68	2.8	0.73	4.2	0.67	4.2

It may again be seen from figures 10 and 11 that the best value is achieved at 120 and 25 iterations respectively. As such there is no further development of the best value after these iterations.

All the situations i.e. Cases (a), (b) and (c) for different noise levels are shown in tables 6 to 8 respectively. It is found from all the cases that maximum error in identification of damage factor is 9% in case of without noise. In case of noise level I and II, the error percentages are 10 and 13 respectively. As such, this methodology can also be applied with noise polluted experimental data.

5.5. Special case

It is to be noted that, the equation for uniform beam with homogenous material may be obtained simply by putting r=0 and e=0 in non-homogenous tapered beam equation for without noise case. The case (c) in the fourth example is solved by putting r=0 and e=0 and compared with the case (b) of the first example. They are found to be in good agreement, which shows the reliability of the model for the non-homogenous tapered beam.

6. CONCLUSION

A procedure has been presented for the simultaneous location and quantification of the damage in a tapered beam with non-homogenous material. Investigation has also been done to study the said problem with and without noise polluted experimental data. Genetic algorithms have been employed for which the optimization function has been formulated in term of modified residual force vectors. The damage factors identified for the beam problem, which are obtained by using GA for optimization purpose, show excellent agreement with those chosen for the mechanical simulation of these damaged structures.

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ZASTOSOWANIE ALGORYTMÓW GENETYCZNYCH DO IDENTYFIKACJI ZNISZCZENIA NIEJEDNORODNYCH STOŻKOWATYCH BELEK

Streszczenie

Niejednorodne elementy strukturalne takie jak belki są istotną częścią konstrukcji inżynierskich i stanowią przedmiot wielu badań doświadczalnych. Nawet niewielkie pęknięcie takiego elementu obniża znacznie jego wytrzymałość i, w konsekwencji, wytrzymałość całej konstrukcji, co może prowadzić do znacznych zniszczeń. Analiza niejednorodnych elementów strukturalnych jest skomplikowana, ponieważ własności materiału zmieniają się w zależności od położenia, a problem staje się nawet bardziej złożony kiedy belka ma kształt stożkowaty. W niniejszej pracy przedstawiono nowe sformułowanie funkcji celu dla optymalizacji takich belek metodą algorytmów genetycznych. Optymalizację połączono z metodą sił residualnych i zastosowano do identyfikacji makroskopowych pęknięć w niejednorodnych belkach o zmiennym przekroju. Opracowany model wymaga danych doświadczalnych jako parametrów wejściowych i pozwala przewidywać lokalizację i rozmiar pęknięcie materiału. Dane wygenerowane numerycznie w oparciu o symulację konstrukcji metodą elementów skończonych zostały wykorzystane do identyfikacji pęknięcia materiału z dobrą dokładnością. Otrzymane teoretycznie parametry zniszczenia zostały porównane z wynikami z opracowanego modelu i otrzymano dobrą zgodność.

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