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FREQUENCY OPTIMIZATION BASED ON "EXACT" SENSITIVITY AND FATIGUE LIFE ESTIMATION

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Abstract

In this paper the optimization of double – disk rotor shaft system based on sensitivity analysis followed by the fatigue life estimation of optimized shaft is considered. The objective of optimization is to avoid resonance, which can cause excessive stresses leading to prematural fatigue cracks. Sensitivities of eigenvalues with respect to design variables are calculated by the Direct Differentiation Method (DDM) (Kleiber et al., 1997; Sosnowski & Kleiber, 1996; Sergeyev & Mroz, 2000). The optimization is performed with the objective to move natural frequencies as far as possible from resonance frequency. Next the shaft system is dynamically loaded. Fluctuation of stress is stored, cycles are counted by rainflow method and next fatigue life estimation is performed. Diameters of selected shaft parts are chosen as design parameters.

Key words: sensitivity analysis, rainflow method, metal fatigue

1. NATURAL FREQUENCY PROBLEM

Lets consider natural frequency problem without dumping (Hien, 1990; Zienkiewicz & Taylor, 2000)

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \tag{1}$$

where **M** and **K** are mass and stiffness matrices respectively, **u** is displacement vector, '·' denotes time derivative.

Assumption is made that the solution of the problem is

$$\mathbf{u}(\tau) = \mathbf{\Phi}\sin\left[\omega\cdot\left(\tau - t_0\right)\right] \tag{2}$$

where ω is frequency and $\mathbf{\Phi}$ is N-dimensional vector of normalized vibration amplitude, τ is time. After differentiation of $\mathbf{u}(\tau)$, substitution into Equation (1) and transformation to global system we obtain the eigenproblem equation

$$\left(\mathbf{K} - \mathbf{\Lambda}\mathbf{M}\right)\tilde{\mathbf{\Phi}} = \mathbf{0} \tag{3}$$

where $\mathbf{\Lambda}$ is diagonal matrix of square natural frequencies ω, $\tilde{\mathbf{\Phi}}$ is global matrix of eigenvectors.

$$\mathbf{\Lambda} = \begin{bmatrix} \omega_1^2 & 0 & \dots & 0 \\ 0 & \omega_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_N^2 \end{bmatrix},$$

$$\tilde{\mathbf{\Phi}} = \left[\mathbf{\Phi}^{(1)} \quad \mathbf{\Phi}^{(2)} \quad \dots \quad \mathbf{\Phi}^{(N)}\right] = \begin{bmatrix} \Phi_1^{(1)} & \Phi_1^{(2)} & \dots & \Phi_1^{(N)} \\ \Phi_2^{(1)} & \Phi_2^{(2)} & \dots & \Phi_2^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_N^{(1)} & \Phi_N^{(2)} & \dots & \Phi_N^{(N)} \end{bmatrix} \tag{4}$$

This is a general linear eigenvalue or characteristic value problem. For non-zero solutions the determinant

$$\left|\mathbf{K} - \mathbf{\Lambda}\mathbf{M}\right| = 0 \tag{5}$$

When stiffness and mass matrices are symmetric and positive defined (all the diagonals of the triangular factors are positive and diagonals are dominating,

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the determinant gives N real and different values of ω^2.

The solution of Equation (5) does not allows to determine actual values of $\mathbf{\Phi}$. Such vectors are known as the normal modes of the system or eigenvectors and are made unique by normalizing. $\tilde{\mathbf{\Phi}}$ is orthogonal matrix, so that (Zienkiewicz & Taylor, 2000)

$$\tilde{\mathbf{\Phi}}^T \mathbf{M} \tilde{\mathbf{\Phi}} = \mathbf{I} \quad (6)$$

calls M-orthonormality and

$$\tilde{\mathbf{\Phi}}^T \mathbf{K} \tilde{\mathbf{\Phi}} = \mathbf{\Lambda} \quad (7)$$

calls K-orthogonality where, **I** is identity matrix.

If we make the system dependent on design parameter vector **h**, eigenvalues and corresponding eigenvectors are also dependent on design parameters vector. In this case we can rewrite the equation (3) as

$$\left(\mathbf{K}(\mathbf{h}) - \mathbf{\Lambda}(\mathbf{h})\mathbf{M}(\mathbf{h})\right)\tilde{\mathbf{\Phi}}(\mathbf{h}) = \mathbf{0} \quad (8)$$

2. EIGENVALUE SENSITIVITY

If the mass and stiffness matrices are symmetric and continuously differentiable with respect to design variable vector **h** and if any eigenvalue ω_i is not repeated, then eigenvalues and corresponding eigenvectors are also continuously differentiable with respect to design variables as was proved by Kleiber et al. (1997) and Hien (1990). After differentiation of Equation (8) we obtain:

$$\frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}} \tilde{\mathbf{\Phi}}(\mathbf{h}) + \mathbf{K}(\mathbf{h}) \frac{\partial \tilde{\mathbf{\Phi}}(\mathbf{h})}{\partial \mathbf{h}} - \frac{\partial \mathbf{\Lambda}(\mathbf{h})}{\partial \mathbf{h}} \mathbf{M}(\mathbf{h})\tilde{\mathbf{\Phi}}(\mathbf{h}) - \mathbf{\Lambda}(\mathbf{h}) \frac{\partial \mathbf{h}(\mathbf{h})}{\partial \mathbf{h}} \tilde{\mathbf{\Phi}}(\mathbf{h}) - \mathbf{\Lambda}(\mathbf{h})\mathbf{M}(\mathbf{h}) \frac{\partial \tilde{\mathbf{\Phi}}(\mathbf{h})}{\partial \mathbf{h}} = \mathbf{0} \quad (9)$$

By leaving $\frac{\partial \mathbf{\Lambda}(\mathbf{h})}{\partial \mathbf{h}}$ factor on the left-hand side, transferring others factors on the right-hand side and putting in order right-hand side factors, we obtain

$$\frac{\partial \mathbf{\Lambda}(\mathbf{h})}{\partial \mathbf{h}} \mathbf{M}(\mathbf{h})\tilde{\mathbf{\Phi}}(\mathbf{h}) = \left[\frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}} - \mathbf{\Lambda}(\mathbf{h}) \frac{\partial \mathbf{M}(\mathbf{h})}{\partial \mathbf{h}}\right]\tilde{\mathbf{\Phi}}(\mathbf{h}) + \left[\mathbf{K}(\mathbf{h}) - \mathbf{\Lambda}(\mathbf{h})\mathbf{M}(\mathbf{h})\right]\frac{\partial \tilde{\mathbf{\Phi}}(\mathbf{h})}{\partial \mathbf{h}} \quad (10)$$

Multiplication of both sides of this equation by $\tilde{\mathbf{\Phi}}^T(\mathbf{h})$ gives

$$\frac{\partial \mathbf{\Lambda}(\mathbf{h})}{\partial \mathbf{h}} \tilde{\mathbf{\Phi}}^T(\mathbf{h})\mathbf{M}(\mathbf{h})\tilde{\mathbf{\Phi}}(\mathbf{h}) = \tilde{\mathbf{\Phi}}^T(\mathbf{h})\left[\frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}} - \mathbf{\Lambda}(\mathbf{h}) \frac{\partial \mathbf{M}(\mathbf{h})}{\partial \mathbf{h}}\right]\tilde{\mathbf{\Phi}}(\mathbf{h}) + \left[\mathbf{K}(\mathbf{h}) - \mathbf{\Lambda}(\mathbf{h})\mathbf{M}(\mathbf{h})\right]\tilde{\mathbf{\Phi}}(\mathbf{h}) \frac{\partial \tilde{\mathbf{\Phi}}(\mathbf{h})}{\partial \mathbf{h}} \quad (11)$$

The second term on the right-hand side is equal zero according to general eigenproblem formulation Equation (8). Using the orthonormality condition Equation (6), we obtain the sensitivity equation

$$\frac{\partial \mathbf{\Lambda}(\mathbf{h})}{\partial \mathbf{h}} = \tilde{\mathbf{\Phi}}^T(\mathbf{h})\left[\frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}} - \mathbf{\Lambda}(\mathbf{h}) \frac{\partial \mathbf{M}(\mathbf{h})}{\partial \mathbf{h}}\right]\tilde{\mathbf{\Phi}}(\mathbf{h}) \quad (12)$$

3. OPTIMIZATION ALGORITHM

The objective of optimization is to move the natural eigenfrequency from the neighborhood of the assumed forced frequency in order to avoid resonance. This may prolong fatigue life of the structure. The forced frequency f_b is related to eigenvalue λ_b by formula (Hien, 1990; Zienkiewicz & Taylor, 2000):

$$\lambda_b = (2\pi f_b)^2 \quad (13)$$

Optimization problem is formulated as follows:

$$c(\mathbf{h}) = \max\left[\left|\lambda - \lambda_b\right|\right] = \min\left[-\left|\lambda - \lambda_b\right|\right], \qquad \lambda \in \mathbf{\Lambda} \quad (14)$$

with constraints:

$$\begin{gathered} h_{\min} \le h_1, h_2, \ldots, h_n \le h_{\max} \\ \left|\mathbf{K}(\mathbf{h}) - \mathbf{\Lambda}(\mathbf{h})\mathbf{M}(\mathbf{h})\right| = 0 \end{gathered} \quad (15)$$

where λ is the closest natural eigenvalue and n is the number of design parameters. Optimization algorithm is based on sensitivity. After differentiation of objective function with respect to one of design parameters h_i we obtain

$$\frac{\partial c(\mathbf{h})}{\partial h_i} = \begin{cases} \dfrac{\partial \lambda(\mathbf{h})}{\partial h_i} & \text{for} \quad \lambda \le \lambda_b \\ -\dfrac{\partial \lambda(\mathbf{h})}{\partial h_i} & \text{for} \quad \lambda > \lambda_b \end{cases} \quad (16)$$

Differentiation of eigenvalue system is performed by Direct Differentiation Method (DDM). In DDM values of $\frac{\partial \lambda(\mathbf{h})}{\partial h_i}$ and all quantities in Equation (12) ($\frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}}, \frac{\partial \mathbf{M}(\mathbf{h})}{\partial \mathbf{h}}$) are calculated accurately. This is most precise and less consuming the processor time (for problems with small number of design variables).

4. THE IDEA OF EQUIVALENT AMPLITUDE STRESS CALCULATIONS FOR DIFFERENT STRESS RATIO

Wöhler S – N curves are obtained from constant amplitude tests of smooth specimens cyclically loaded between a maximum (S_{max}) and minimum

(S_{min}) stress level. Load parameter, which has largest influence to fatigue life is the stress amplitude given by formula

$$S^a = \frac{S_{\max} - S_{\min}}{2} \qquad (17)$$

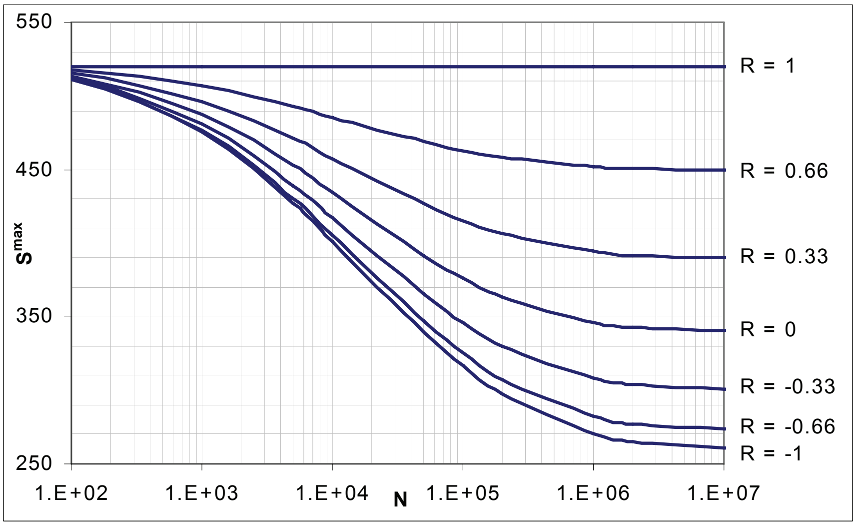

Fig.1. Wöhler S-N curves for different stress ratios.

The mean stress (S^m) must also be taken into consideration. The mean stress effect can be taken into account by definition of the stress ratio R as a relation between maximum and minimum stress value.

$$R = \frac{S_{\min}}{S_{\max}} \qquad (18)$$

The influence of stress ratio on the fatigue life of the structure is presented in figure 1 (N is number of cycles).

Creation of fatigue band slips, and next fatigue crack propagation depend only on extreme values of stresses (S_{max} and S_{min}). The path between consecutive S_{max} and S_{min} has no influence to the fatigue life, under condition that the path is monotonic and there are not other cycles between S_{max} and S_{min}. We are assuming a sinusoidal load scheme with given stress amplitude S_a and stress ratio R.

The objective of this section is to develope the algorithm of calculation of the amplitude stress $S^{ae}{}_{R=-1}$, which is *equivalent* to the given value $S^a{}_{R\neq-1}$ (where $S^m \neq 0$). The assumption is made that such amplitude stress $S^{ae}{}_{R=-1}$ gives the same number of cycles to fatigue failure as it is for given $S^a{}_{R\neq-1}$. This assumption is based on Goodman and Wöhler curves dependency presented in figure 2. Further the amplitude stress $S^a{}_{R\neq-1}$ can be obtained from the finite element analysis of specific structure at hand loaded by arbitrary non-symmetric load, $R \neq -1$.

When specific fatigue data are missed, for example, number cycles to failure for the given stress ratio, one can use, in some range, empirical relation between amplitude stress and fatigue life N as a linear approximation of the $S-N$ curve in log-log coor-

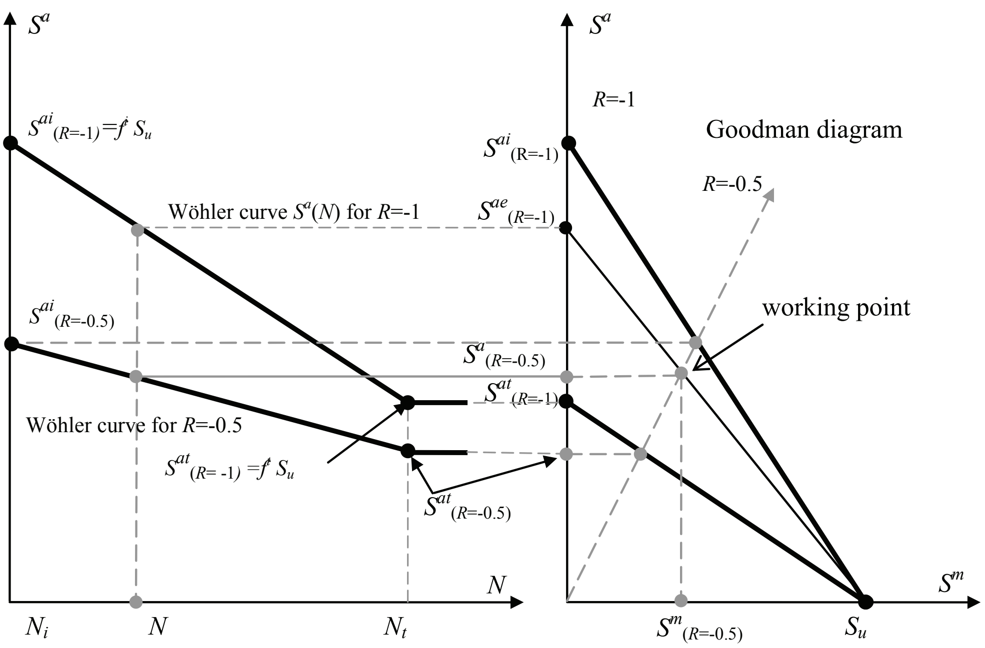

Fig. 2. Wöhler and Goodman diagrams dependency.

dinates. This range is described by two points: initial amplitude stress (S^{ai}) at about 1000 cycles and threshold stress (S^{at}) at approximately 2 millions cycles. Such linear Wöhler *S-N* curve for steel is shown on the left side of figure 2 and can be represented (for $R = -1$) by the equation

$$S^a(N) = 10^{\frac{\log\left[S^{at}_{(R=-1)}\right] - \log\left[S^{ai}_{(R=-1)}\right]}{\log(N_t) - \log(N_i)}\left[\log(N) - \log(N_i)\right] + \log\left[S^{ai}_{(R=-1)}\right]} \tag{19}$$

where $N_i = 1e3$ and $N_t = 2e6$.

Substituting the amplitude stress at the considered point of the structure by value $S^a(N) = S^a_{(R=-1)}$ and modifying the equation (19) we obtain the number of cycles to failure N

$$N = 10^{\frac{\log\left[S^a_{(R=-1)}\right]\log(N_i) - \log\left[S^a_{(R=-1)}\right]\log(N_t) + \log(S^{ai})\log(N_t) - \log(S^{at})\log(N_i)}{\log(S^{ai}) - \log(S^{at})}} \tag{20}$$

In this case $S^a_{(R=-1)} = S^{\max}$ due to the assumption of $R = -1$ load scheme.

Wöhler curves are obtained for the load scheme $R = -1$ by bending fatigue tests and rarely the axial-load tests ($R = 0$). However these $R = 0$ or $R = -1$ mean stress ratio is not typical for real industrial components working under cyclic load.

Using the value of ultimate stress S_u of the material and a value of the amplitude stress for different $R \neq -1$ we can obtain the *equivalent* value of the amplitude stress for $R = -1$ from Goodman diagram (see figure 2 right). Then this value can be applied in the equation (20) in order to calculate number of cycles to failure N.

Goodman diagram (figure 2) is represented by the line between amplitude stress $S^a_{(R=-1)}$ (S^m equals 0) and ultimate stress of the material on S^m axis. This relation is given by equation

$$S^a_{(R\neq-1)} = S^{ae}_{(R=-1)}\left[1 - \left(\frac{S^m}{S_u}\right)^n\right] \tag{21}$$

When $n = 1$ the equation (21) is Goodman equation, at $n = 2$ we obtain Gerber equation. This relationship permits to obtain the equivalent amplitude stress S^{ae} for any load scheme.

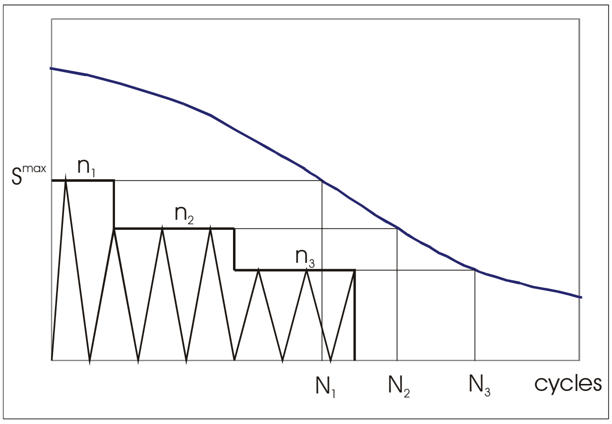

Fig. 3. Idea of fatigue life estimation by damage accumulation hypothesis.

From the typical finite element analysis we can get maximal stress at the most stressed point of the structure.

The value of the mean stress S^m in the considered structure point can be obtained from the equation

$$S^m = \frac{S^{\max}(1+R)}{2} \tag{22}$$

where $S^{\max}$ is a maximal stress value obtained for any R from FEM analysis. Then $S^a_{(R\neq-1)}$ are calculated from the following relations

$$S^a_{(R\neq-1)} = S^{\max} - S^m \tag{23}$$

These values are used in the modified equation (21)

$$S^{ae}_{(R=-1)} = \frac{S^a_{(R\neq-1)}}{1 - \left(S^m / S_u\right)^n} \tag{24}$$

After substituting the values from the equations (22) and (23) into the equation (24) we obtain the equivalent amplitude stress as a function of a maximal stress value from FE analysis for any stress ratio R

$$S^{ae}_{(R=-1)} = \frac{-0,5\,S^{\max}_{(R\neq-1)}(R+1) + S^{\max}_{(R\neq-1)}}{1 - \left[\frac{1}{2}(R+1)\frac{S^{\max}_{(R\neq-1)}}{S_u}\right]^n} \tag{25}$$

When we use Goodman relationship ($n = 1$) the equation (25) takes more simpler

$$S^{ae}_{(R=-1)} = \frac{S^{\max}_{(R\neq-1)}(1-R)S_u}{2S_u - S^{\max}_{(R\neq-1)}(R-1)} \tag{26}$$

Then the value of the equivalent amplitude stress calculated from the equations (25) or (26) is substituted into the equation (20), where $S^a_{(R=-1)} \equiv S^{ae}_{(R=-1)}$, in order to calculate the number of cycles to failure of the analyzed structure loaded by arbitrary non-symmetric load with any stress ratio $R \neq -1$.

The ideas of fatigue life estimation of structures, which inertial forces are comparable to external load are to calculate a damage caused by a single cycle and damage accumulation hypothesis (see figure 3). A accumulated damage caused by all cycles in block can be calculated by formula (Jakubczak, 2002)

$$d = \sum_{i=1}^{k} \frac{n_i}{N_i} \quad (27)$$

where n_i is number of cycles of each stress level and N_i is number of cycles to failure with given stress level.

Fatigue durability may be expressed by number of cycles to expected failure N or hours of safety work of the structure T given by formulas (Jakubczak, 2002)

$$N = \frac{D}{d} \cdot n_p \qquad T = \frac{D}{d} \cdot t_w \quad (28)$$

where n_p is number of cycles in given time interval, t_w is time of work interval and D is assumed critical damage value. In this paper critical damage value is obtained from relation

$$D = \frac{\sigma_{\max}}{S_u} \quad (29)$$

where σ_{max} is maximum stress values in structure and S_u is ultimate stress of material.

parts, l_1 = 370 mm, l_2 = 50 mm, l_3 = 80 mm, l_4 = 220 mm, l_5 = 100 mm, l_6 = 100 mm, l_7 = 200 mm, l_8 = 200 mm, l_9 = 200 mm, l_{10} = 100 mm, l_{11} = 100 mm, l_{12} = 220 mm, l_{13} = 80 mm, l_{14} = 155 mm, l_{15} = 145 mm respectively.

Material of the shaft is steel with Young modulus $E = 210$ *GPa*, Poisson's ratio is $\nu = 0.3$, density $\rho = 7850\ kg/m^3$ and ultimate stress $S_u = 520$ *MPa*. Both rigid disks are the same andhave parameters: mass $m_1 = m_2 = 300\ kg$ and diametral mass moment of inertia $J_1 = J_2 = 3.176\ kg/m^3$. Both bearings are the same too, stiffenesses are: $k_{xx} = 3.15$ *kPa*, $k_{yy} = -3.15$ *kPa*, $k_{xy} = 1.3$ *MPa and* $k_{yx} = -1.3$ *MPa*. The design parameters are diameters of parts number 7, 8 and 9. Initial values are $\boldsymbol{\phi} = \phi_7 = \phi_8 = \phi_9 = 80\ mm$. These diameters may vary between value of *50 mm* and *180 mm*. Lower limit restricts maximal deflection of the shaft, upper limit is assumed due to maximum weight of shaft system. The objective function may be rewritten from Equation. (14) and (15) as:

$$c(\boldsymbol{\varphi}) = \max\left[\left|\lambda_b - \lambda\right|\right] = \min\left[-\left|\lambda_b - \lambda\right|\right] \quad (30)$$

with constraints:

$$\begin{gathered} 50 \le \phi_7, \phi_8, \phi_9 \le 180 \\ \left|\mathbf{K}(\boldsymbol{\varphi}) - \boldsymbol{\lambda}(\boldsymbol{\varphi})\mathbf{M}(\boldsymbol{\varphi})\right| = 0 \end{gathered} \quad (31)$$

Rotation speed of shaft *3000 rpm* corresponds to oscillating forced load with frequency *50 Hz*. This case is typical in electrical generators.

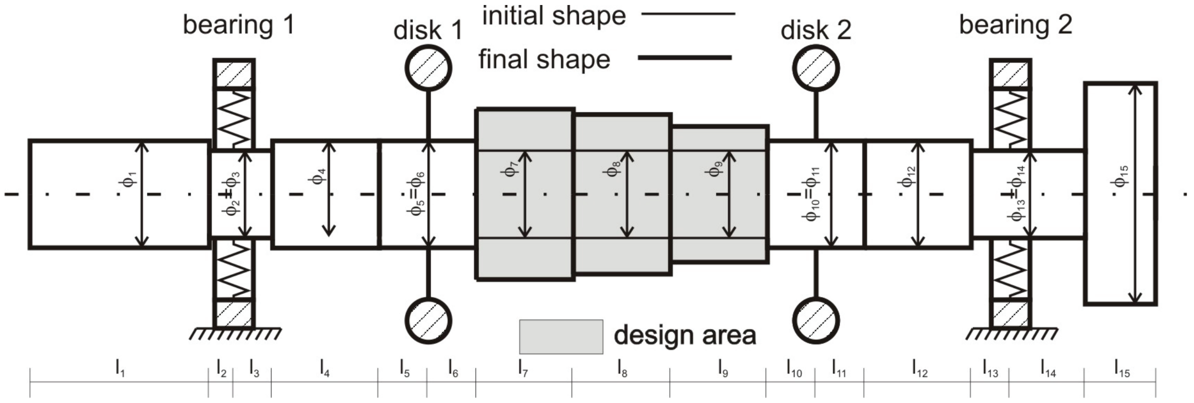

Fig. 4. *Double – disk rotor shaft system before and after optimization.*

5. NUMERICAL EXAMPLES

The double – disk shaft system presented in figure 4 is considered. Dimensions are as follows: diameters, ϕ_1 = 90 mm, $\phi_2 = \phi_3$ = 80 mm, $\phi_4 = \phi_5 = \phi_6$ = 90 mm, $\phi_7 = \phi_8 = \phi_9$ = 80 mm, $\phi_{10} = \phi_{11} = \phi_{12}$ = 90 mm, $\phi_{13} = \phi_{14}$ = 80 mm, ϕ_{15} = 152 mm and length of

Consecutive optimization steps are presented in figure 5. Sensitivities are calculated by Direct Differentiation Method. Sensitivity charts with respect to ϕ_7, ϕ_8 and ϕ_9 are presented in figure 6. Optimization is done in *13* steps. Final values of design parameters are: $\phi_7 = 135.5\ mm$, $\phi_8 = 132.6\ mm$ and ϕ_9

= *122.9 mm*. We can observe in figure 5 that objective function was improved by about *14 Hz*.

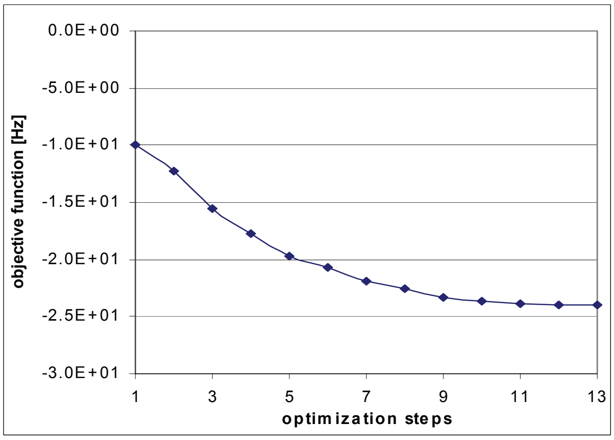

Fig. 5. *Optimization path.*

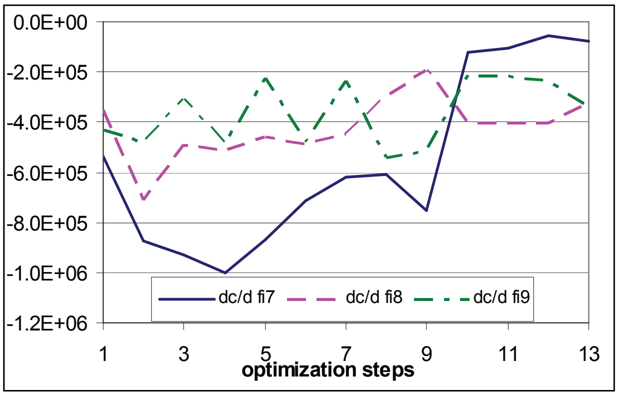

Fig. 6. *Sensitivities of the objective function with respect to design parameters.*

algorithm, which is integral part of Matlab Wave Analysis Toolbox, was used.

Maximum stress value noticed in shaft system before optimization is *247.2 MPa* and corresponds to about 253 hours of work up to expected failure. Fatigue failure of the shaft system are eliminated due to the optimization. Maximum stress value is equal 36.2 MPa and it is below fatigue limit.

6. CONCLUSIONS

- Present work deals with optimization performed on the basis of exact sensitivity.
- Optimization with objective to avoid resonance allows to eliminate undesirable vibration, noise and can increase fatigue life.
- Equivalent amplitude stress is very effective tool in fatigue analysis in engineering cases, when fatigue data are provided only for stress ratio $R = -1$, but also in dynamic fatigue analysis where rainflow algorithm is used.
- Obtained fatigue life of the double – disk shaft system is theoretical and requires experiment confirmation.

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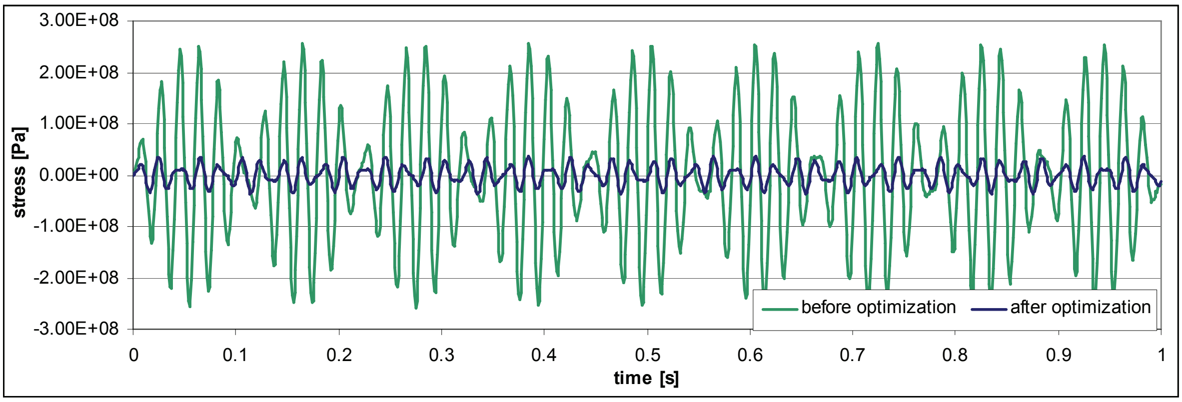

Fig. 7. *Stress fluctuations in shaft system before and after optimization.*

To check the influence of optimization, fatigue analysis of shaft system was performed. Rotating shaft with speed *3000 rpm* was numerically tested. Stress fluctuations in shaft system before and after optimization are calculated. The comparison of stress fluctuations in most stressed point before and after optimization are presented in figure 7.

To perform fatigue analysis specified counting algorithm must be chosen. The rainflow method

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OPTYMALIZACJA CZĘSTOTLIWOŚCI WŁASNYCH Z WYKORZYSTANIEM ANALIZY WRAŻLIWOŚCI ORAZ OCENA WYTRZYMAŁOŚCI ZMĘCZENIOWEJ

Streszczenie

Przedmiotem artykułu jest optymalizacja ze względu na częstotliwości drgań własnych dowolnych konstrukcji z wykorzystaniem ścisłych metod analizy wrażliwości. Zbadany został wpływ optymalizacji na wytrzymałość zmęczeniową. Celem optymalizacji było uniknięcie rezonansu, który może prowadzić do wzrostu naprężeń w konstrukcji i przedwczesnego zniszczenia zmęczeniowego. Wrażliwość wartości własnych ze względu na parametry projektowe została obliczona Metodą Bezpośredniego Różniczkowania (Direct Differentiation Method, DDM) wcześniej rozwijaną przez Sergeyeva i Mroza, (2000), Kleibera i innych (1997) oraz Sosnowskiego i Kleibera (1996). Jako przykład wybrany został wał wirnikowy z dwoma dyskami sztywnymi, podparty na dwóch łożyskach ślizgowych. Parametrami projektowymi były średnice poszczególnych sekcji wału. Proces optymalizacji miał na celu oddalenie częstotliwości drgań własnych wału tak daleko jak to możliwe (przy zachowaniu odpowiednich ograniczeń) od założonej częstotliwości wymuszenia. Następnie optymalny wał został obciążony siłą zmienną w czasie. Zmiany naprężenia w czasie zostały zapisane, cykle naprężeń zostały zliczone algorytmem „rainflow”. Następnie dokonano porównawczej analizy zmęczeniowej wału przed i po optymalizacji.

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